1 st Sem Math's I

Basic Mathematics (BM)

Unit - I - Matrices

Unit II = Trigonometry and Analytical Geometry

Unit III = Differential calculus

Unit IV = Integral Calculus

Monday
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Numbers

* Numbers - There are seven types of numbers.

(1) Natural Numbers :- Numbers starting from 1 are called natural no.
This are denoted by N i.e $N = 1, 2, 3, 4 \ldots\ldots$

(2) Whole Numbers :- The numbers starting from zero called whole numbers.
This are denoted by W i.e $W = 0, 1, 2, 3, 4 \ldots\ldots$

(3) Integers Numbers :- The ~~numbers~~ ~~Starting~~ from positive numbers and negative numbers including zero are called Integers
This are denoted by Z i.e $Z = \ldots\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4 \ldots$

(4) Rational Numbers :- The numbers ~~which~~ in the form of $\frac{p}{q}$ where $q \neq 0$ are called rational no.
This are denoted by $\mathbb{Q}$ i.e $\mathbb{Q} = \frac{3}{1}, \frac{2}{4}, \frac{4}{2}, \ldots\ldots$

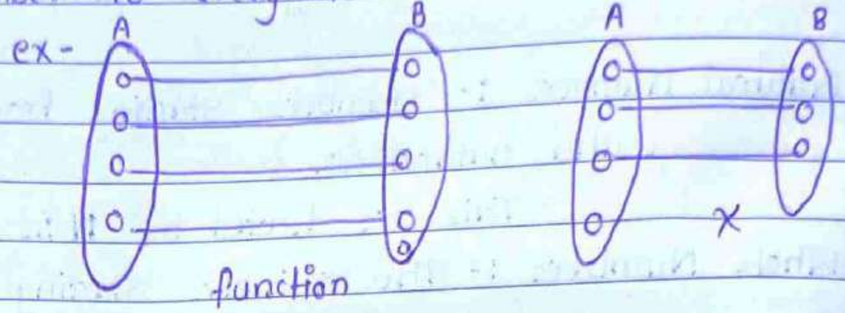
(5) Irrational Numbers :- The numbers in the radical form or in root form are called irrational no.
This are denoted by $\mathbb{Q}^c$ i.e $\mathbb{Q}^c = \sqrt{9}, \sqrt{16}, \ldots\ldots \sqrt{a}, \sqrt{b}, \sqrt{c}$.

(6) Real Numbers :- The collection of rational & irrational numb are called real numbers.
This are denoted by R i.e $R = \{\mathbb{Q}, \mathbb{Q}^c\}$

(7) Complex Numbers :- The no. in the form of $a+ib$ are called complex numbers.
This are denoted by C i.e $C = a+ib$; $a, b \in R$

1) Function :- f(x) A $f: A \to B$ is the assignment of unique number to every element of A in B.

ex -



2) Domain :- if $f: A \to B$ is a function, then set A is called domain. In a function every element of domain is assigned.

3) Co-domain :- If $f: A \to B$ is a function, the set B is called co-domain

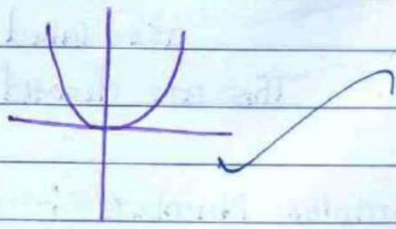
4) Range :- If $f: A \to B$ is a function then, the subset of B ~~is~~ which is assinged with A is called range.

5) Even function :- The f(x) is said to be even function if $f(-x) = x$

In even function resultant is positive & its graph is symmetric about Y-axis

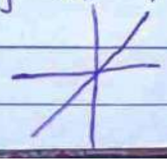
eg - $f(x) = x^2$

$f(x) = \cos x$



6) Odd function :- The f(x) is said to be odd function if $f(-x) = -x$

In odd function resultant is negative & its graph is symmetric about origin



eg - $f(x) = x^3$

$f(x) = \sin x$

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Unit-1. Matrices

Basic r

1) Matrix :- Regular arrangement of numbers in a rows and columns enclosed by [ ], is known as matrix. The no. of rows and columns in a matrix is known as its order. generally the matrix is given as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ - & - & - & - & - \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$

2) Row matrix :- The matrix having one / single row and any no. of columns is known as row matrix.

eg- $R = \begin{bmatrix} 1 & 5 & 3 & 0 \end{bmatrix}$

order of $R = 1 \times 4$

generally the matrix is given as $A_{1 \times n}$ is

3) Column matrix :- The matrix having one / single column and any no. of rows is known as column matrix.

eg- $C = \begin{bmatrix} 1 \\ 5 \\ 3 \\ 0 \end{bmatrix}$ generally the matrix is given as $A_{n \times 1}$

Order of $C = 4 \times 1$

4) Square matrix :- The matrix having same no. of rows and columns is known as square matrix

generally $A_{n \times m}$ OR A_n is a square matrix

eg- $A_n = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ - & - & - & - & - \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix}$

$P \times I = P$
$P + 0 = P$

5) Rectangular matrix :- The matrix in which either rows are greater than columns or cloumns greater than rows is known as Rectangular matrix
generally $A_{n \times m}$, $n > m$
or $m > n$

6) Null / zero matrix :- The matrix in which all elements are equal to zero is known Null/zero matrix.
generally

$$O = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & 0 \dots & 0 \end{bmatrix}$$

7) Unit / Identity matrix :- The matrix in which all the diagonal elements are equal to ~~one~~ non zero ~~or~~ unity and equal to one and rest all zero.
generally

$$I = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ - & - & - & - \\ 0 & 0 & \dots & 1 \end{bmatrix} \text{ OR } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

8) Diagonal matrix :- Any square matrix whose diagonal elemensts are non zero and rest of the elements are zero.

generally $D_n = \begin{bmatrix} d_1 & 0 & 0 & \dots & 0 \\ 0 & d_2 & 0 & \dots & 0 \\ 0 & 0 & \dots & \dots & d_n \end{bmatrix}$

9) Scalar matrix :- The diagonal matrix whose non zero elements are Same / equal is known as Scalar ~~Square~~ matrix.
generally

$$S_n = \begin{bmatrix} a & 0 & 0 & \dots & 0 \\ 0 & a & 0 & \dots & 0 \\ 0 & 0 & \dots & \dots & a \end{bmatrix}$$

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10) Transpose of a matrix :- The transpose of matrix A is obtained by interchanging its rows and columns. It is denoted by A^T or A'. generally if

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ - & - & - & - & - \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$

then

$$A^T = \begin{bmatrix} a_{11} & a_{21} & a & a_{m1} \\ a_{12} & a_{22} & \vdots & a_{m2} \\ a_{13} & a_{23} & \vdots & a_{m3} \\ \vdots & \vdots & & \\ a_{1n} & a_{2n} & \vdots & a_{mn} \end{bmatrix}$$

* Find the transpose of following matrices.

1) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 1 & 5 \end{bmatrix} \longrightarrow A' = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 4 & 5 \end{bmatrix}$

2) $B = \begin{bmatrix} -1 & 0 & 4 \\ 5 & 10 & -1 \\ 4 & -11 & 9 \end{bmatrix} \longrightarrow B' = \begin{bmatrix} -1 & 5 & 4 \\ 0 & 10 & -11 \\ 4 & -1 & 9 \end{bmatrix}$

11) a) Symmetric matrix :- The matrix A is said to be Symmetric matrix if $A' = A$

eg - $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 7 \end{bmatrix}$

$\therefore A' = A$

b) Skew Symmetric matrix :- The matrix B is said to be skew symmetric if $B^T = -B$, where the diagonal elements are zero.

i.e. $a_{ij} = 0$

$a_{ij} = 0 \ \forall \ i = j$

eg - $B = \begin{bmatrix} 0 & 2 & -5 \\ -2 & 0 & 7 \\ 5 & -7 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & -11 & -15 \\ 11 & 0 & 12 \\ 15 & -12 & 0 \end{bmatrix}$

12) Matrix Addition :- Two matrixes A and B can be added if order is same and corresponding elements will be added.

eg. $A = \begin{bmatrix} 3 & 2 & -5 \\ -2 & 4 & 7 \\ 5 & -7 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 7 & -11 & -15 \\ 11 & 8 & 12 \\ 15 & -12 & 9 \end{bmatrix}$

$A + B = \begin{bmatrix} 3 & 2 & -5 \\ -2 & 4 & 7 \\ 5 & -7 & 5 \end{bmatrix} \begin{bmatrix} 7 & -11 & -15 \\ 11 & 8 & 12 \\ 15 & -12 & 9 \end{bmatrix} = \begin{bmatrix} 3+7 & 2+(-11) & -5+(-15) \\ -2+11 & 4+8 & 7+12 \\ 5+15 & -7+(-12) & 5+9 \end{bmatrix}$

$= \begin{bmatrix} 10 & -9 & -20 \\ 9 & 12 & 19 \\ 20 & -19 & 14 \end{bmatrix}$

13) Multiplication Matrix :- The multiplication matrix is only possible when 1^{st} matrix columns is equal to 2^{nd} matrix row.

i.e. $A = [a_{ij}]_{m \times n}$

$B = [b_{ij}]_{n \times p}$

Then AB is a matrix of order $m \times p$.

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eg. If $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 1 & 2 \end{bmatrix}_{2\times 3}$

$B = \begin{bmatrix} 5 & 3 & 2 \\ 1 & 5 & 4 \\ 3 & 1 & 2 \end{bmatrix}_{3\times 3}$

$\therefore AB = 2\times 3$

$$A = \begin{bmatrix} 10+3+12 & 6+15+4 & 4+12+8 \\ 25+1+16 & 15+5+2 & 10+4+4 \end{bmatrix} = \begin{bmatrix} 25 & 25 & 24 \\ 32 & 22 & 28 \end{bmatrix}_{2\times 3}.$$

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14) Scalar multiplication :- Is the multiplication of number to the every element of matrix.
generally, it is denoted by
i.e - KA

$$KA = \begin{bmatrix} Ka_{11} & Ka_{12} & \cdots & Ka_{1n} \\ Ka_{21} & Ka_{22} & \cdots & Ka_{2n} \\ \vdots & \vdots & & \vdots \\ Ka_{m1} & Ka_{m2} & \cdots & Ka_{mn} \end{bmatrix}$$

Ex: If $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 5 & 2 \\ -1 & 0 & 4 \end{bmatrix}$ find 7A

Soln:- $7A = 7\begin{bmatrix} 2 & 1 & 3 \\ 1 & 5 & 2 \\ -1 & 0 & 4 \end{bmatrix}$

$$\Rightarrow 7A = \begin{bmatrix} 14 & 7 & 21 \\ 7 & 35 & 14 \\ -7 & 0 & 28 \end{bmatrix}$$

15) Determinant of a matrix :- The determinant of a matrix is a real no. and is denoted as
$|A|$

if $|A| = 0$, then A is said to be a singular matrix
if $|A| \neq 0$, then A is said to be a nonsingular matrix

Ex- $A = \begin{bmatrix} 2 & 5 \\ 2 & 5 \end{bmatrix}$ $\therefore |A| = 0$, 'A' is Singular

$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ $\therefore |A| \neq 0$, 'A' is non singular

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16) Minor and Cofactor of an element

Consider a matrix

1)

$$A_3 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\Rightarrow |A_3| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

minor of $a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

minor of $a_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

minor of $a_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

Similarly, we can find minors of ~~aa~~ $a_{21}, a_{22}, a_{23}, a_{31}, a_{32}$, and a_{33}.

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Now,

Cofactor of $a_{11} = (-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

Cofactor of $a_{12} = (-1)^{1+2} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = - \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

Cofactor of $a_{13} = (-1)^{1+3} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

Ex- find the minor and co-factor of following matrices.

(1) $A = \begin{bmatrix} 1 & 5 \\ 3 & 2 \end{bmatrix}$ (2) $B = \begin{bmatrix} -1 & 0 & -1 \\ 2 & -1 & 5 \\ 3 & 0 & -3 \end{bmatrix}$

1st → solution - $A = \begin{bmatrix} 1 & 5 \\ 3 & 2 \end{bmatrix}$

cofactor of 1 = 1
Cofactor of 5 = -3
Cofactor of 3 = -5
Co-factor of 2 = 1

$|A| = \begin{vmatrix} 1 & 5 \\ 3 & 2 \end{vmatrix}$

minor of 1 = 2
minor of 5 = 3
minor of 3 = 5
minor of 2 = 1

2nd → solution - $B = \begin{bmatrix} -1 & 0 & -1 \\ 2 & -1 & 5 \\ 3 & 0 & -3 \end{bmatrix}$ $|B| = \begin{vmatrix} -1 & 0 & -1 \\ 2 & -1 & 5 \\ 3 & 0 & -3 \end{vmatrix}$

Cofactors $-1 = (-1)^{1+1} \begin{vmatrix} -1 & 5 \\ 0 & -3 \end{vmatrix} = 3 - 0 = \underline{\underline{3}}$

Cofactors $0 = (-1)^{1+2} \begin{vmatrix} 2 & 5 \\ 3 & -3 \end{vmatrix} = -6 - 15 = \underline{\underline{-21}}$

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Co factor of -1 $= (-1)^{1+3}\begin{vmatrix} 2 & -1 \\ 3 & 0 \end{vmatrix} = \underline{3}$ co-factor of

Co-factor of $2 = (-1)^{2+1}\begin{vmatrix} 0 & -1 \\ 0 & -3 \end{vmatrix} = 0$

Co-factor of $-1 = (-1)^{2+2}\begin{vmatrix} -1 & -1 \\ 3 & -3 \end{vmatrix} = 6$

Co-factor of $5 = (-1)^{2+3}\begin{vmatrix} -1 & 0 \\ 3 & 0 \end{vmatrix} = 0$

17) Adjoint and Inverse of a matrix - The transpouse matrix of co-factor matrix is Known as adjoint of matrix. If A is a non-singular matrix then its inverse exist and can be written as $A^{-1} = \frac{\text{adj}A}{|A|}$

[Note - Inverse of a singular matrix does not exist]

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18) Properties of determinants

(1) If any two rows or columns of a determinant are interchanged then the value of determinant remains Same but differ in sign.

If $|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = G$

$\Rightarrow |A'| = \begin{vmatrix} d & e & f \\ a & b & c \\ g & h & i \end{vmatrix} = -G$

(ii) If any two rows or columns of a determinant are identical then the value of determinant is zero

i.e

$$|A| = \begin{vmatrix} a & b & c \\ a & b & c \\ d & e & f \end{vmatrix} = 0$$

(iii) If any row or column is multiplied by a scalar K then the value of determinant is also multiplied by a scalar K.

i.e

$$\text{If } |A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = G$$

$$\text{then } |A^\circ| = \begin{vmatrix} Ka & Kb & Kc \\ d & e & f \\ g & h & i \end{vmatrix} = KG$$

(iv) If ~~any~~ ~~to~~ the elements in any row or column are expressed as sum of two elements. then the determinant can be expressed as sum of two determinant.

$$\text{i.e. If } |A| = \begin{vmatrix} a+b & c+d & e+f \\ g & h & i \\ j & K & l \end{vmatrix}$$

$$\text{then } |A| = \begin{vmatrix} a & c & e \\ g & h & i \\ j & K & l \end{vmatrix} + \begin{vmatrix} b & d & f \\ g & h & i \\ j & K & l \end{vmatrix}$$

(v) If the elements of any row or column is multiplied by a Scalar k and added to the corresponding elements of other row or column, then the value of determinant doesn't alter.

example - If $|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = G$

then

$$|A'| = \begin{vmatrix} a+kd & b+ke & c+kf \\ d & e & f \\ g & h & i \end{vmatrix} = G$$

19) Some Important examples of Inverse of a matrix.

Ex 1) find the Inverse of

$A = \begin{bmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{bmatrix}$ cosider $|A| = \begin{vmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{vmatrix}$

$$\begin{array}{r} 27 \\ -18 \\ \hline 9 \end{array}$$

$$|A| = 1(-5+8) - 4(-2-4) + (-2)(4+5)$$
$$= 1(3) - 4(-6) - 2(9)$$
$$= 3+24-18$$
$$= 27-18 = 9 \neq 0$$

Since $|A| \neq 0$

$\therefore A^{-1}$ exist

Co-factors of matrix

~~A)~~ $A_{11} = (-5+8)$
$A_{11} = \underline{\underline{3}}$

$A_{12} = 4 = \begin{vmatrix} -2 & 4 \\ 1 & 1 \end{vmatrix} = -2-4.$
$= \underline{\underline{6}}$

$A_{13} = -2 = \begin{vmatrix} -2 & -5 \\ 1 & -2 \end{vmatrix} = 4+5 = \underline{\underline{9}}$

$A_{21} = -2 = \begin{vmatrix} 4 & -2 \\ -2 & 1 \end{vmatrix} = 4-4 = \underline{\underline{0}}$

~~A_{22}~~ Co factor of $-5 = \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} = 1+2 = \underline{\underline{3}}$

Co factor of $4 = (-1)^{2+3} \begin{vmatrix} 1 & 4 \\ 1 & -2 \end{vmatrix} = -2-4 = -6 = \underline{\underline{6}}$

Co factor of $1 = (-1)^{3+1} \begin{vmatrix} 4 & -2 \\ -5 & 4 \end{vmatrix} = 16-10 = \underline{\underline{6}}$

Co-factor of $-2 = (-1)^{3+2} \begin{vmatrix} 1 & -2 \\ -2 & 4 \end{vmatrix} = 4-4 = \underline{\underline{0}}$

Co-factor of $1 = (-1)^{3+3} \begin{vmatrix} 1 & 4 \\ -2 & -5 \end{vmatrix} = -5+8 = \underline{\underline{3}}$

Therefore matrix of a Co-factor $= \begin{bmatrix} 3 & 6 & 9 \\ 0 & 3 & 6 \\ 6 & 0 & 3 \end{bmatrix}$

Now adj A $= \begin{bmatrix} 3 & 0 & 6 \\ 6 & 3 & 0 \\ 9 & 6 & 3 \end{bmatrix}$

$\therefore A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{9}\begin{bmatrix} 3 & 0 & 6 \\ 6 & 3 & 0 \\ 9 & 6 & 3 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 & 2/3 \\ 2/3 & 1/3 & 0 \\ 1 & 2/3 & 1/3 \end{bmatrix}$

Q1) find the inverse of following matrices

1) $A = \begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}$ 2) $B = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$

Soln of 1 $\longrightarrow$ $|A| = \begin{vmatrix} 1 & 2 \\ 3 & 8 \end{vmatrix} = 8-6$
$= \underline{\underline{2}}$

$|A| \neq 0$

$\therefore A^{-1}$ exist

Co-factors of matrix $= \begin{bmatrix} 8 & -3 \\ -2 & 1 \end{bmatrix}$ $(-1)^{1+2}$

adj A $= \begin{bmatrix} 8 & -2 \\ -3 & 1 \end{bmatrix}$

$\therefore A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{2}\begin{bmatrix} 8 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -3/2 & 1/2 \end{bmatrix}$

2) $B = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix} \longrightarrow$ $|B| = 1(-28+30) - 0 - 1(-18-0)$

$= 1(2) - 1(-18)$

$= 2 + 18 = 20$

$\therefore 20 \neq 0$

$\therefore |B| = 0$

B^{-1} exist

Co-factor of matrix

Cofactor of $1 = (-1)^{1+1} \begin{vmatrix} 4 & 5 \\ -6 & -7 \end{vmatrix} = -28+30 = \underline{\underline{2}}$

Co factor of $0 = (-1)^{1+2} \begin{vmatrix} 3 & 5 \\ 0 & -7 \end{vmatrix} = -21-0 = -21 = \underline{\underline{21}}$

Co-factor of $-1 = (-1)^{1+3} \begin{vmatrix} 3 & 4 \\ 0 & -6 \end{vmatrix} = -18-0 = \underline{\underline{-18}}$

Co-factor of $3 = (-1)^{2+1} \begin{vmatrix} 0 & -1 \\ -6 & -7 \end{vmatrix} = 0-6 = -6 = \underline{\underline{6}}$

Co-factor of $4 = (-1)^{2+2} \begin{vmatrix} 1 & -1 \\ 0 & -7 \end{vmatrix} = -7-0 = \underline{\underline{-7}}$

Co factor of $5 = (-1)^{2+3} \begin{vmatrix} 1 & 0 \\ 0 & -6 \end{vmatrix} = -6-0 = \underline{\underline{6}}$

Co-factor of $0 = (-1)^{3+1} \begin{vmatrix} 0 & -1 \\ 4 & 5 \end{vmatrix} = 0+4 = \underline{\underline{4}}$

Co-factor of $-6 = (-1)^{3+2} \begin{vmatrix} 1 & -1 \\ 3 & 5 \end{vmatrix} = 5+3 = \underline{\underline{8}}$

Co-factor of $-7 = (-1)^{3+3} \begin{vmatrix} 1 & 0 \\ 3 & 4 \end{vmatrix} = 4-0 = \underline{\underline{4}}$

Co factor matrix $= \begin{bmatrix} 2 & 21 & -18 \\ 6 & -7 & 6 \\ 4 & 8 & 4 \end{bmatrix}$ $\therefore$ Adj $B = \begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & 8 \\ -18 & 6 & 4 \end{bmatrix}$

Inverse of of matrix $= \dfrac{\text{Adj}\,B}{|B|} = \dfrac{1}{20} \begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & 8 \\ -18 & 6 & 4 \end{bmatrix}$

$$B^{-1} = \begin{bmatrix} 1/10 & 3/10 & 1/5 \\ 21/20 & -7/20 & 2/5 \\ -9/10 & 3/10 & 1/5 \end{bmatrix}$$

Ex 3) Verify the orthogonality

$$A = \begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$$

$\rightarrow$ Soln.- To verify orthogonality

$$AA' = I$$

Consider $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $-9+16$

$$\begin{bmatrix} 1+4 & -3-8 \\ -3-8 & 9+16 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -11 \\ -11 & 25 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore$ It is not orthogonal.

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20) Echlon Matrix - The matrix A is said to be echlon form of a matrix if it satisfy the following condition.

(1) first entry should be one (1)
(2) The second row below the first entry should be 0 followed by 1.
(3) the last row should be either 0 or ~~one~~ 1 followe by the zero.

example - $A = \begin{bmatrix} 1 & 3 & 2 & -1 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

The rank of an echlon matrix is equal to no. of non-zero row

* Any matrix can be reduced to echlon form by making the use of row transformation.

* Reduce into the echlon form & find the rank

$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

$R_2 \longrightarrow R_2 - R_1$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 2 & 6 & 5 \end{bmatrix}$

$R_2 \longrightarrow \frac{R_2}{2}$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1/2 \\ 2 & 6 & 5 \end{bmatrix}$

$R_3 \longrightarrow \cancel{R_3 \times R_2}$ $\frac{0}{R_3}$ $\cancel{\frac{3 \times 1}{2}}$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix}$

~~Rank = 2~~

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1) $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

$R_2 \rightarrow R_2 - R_1$

$A \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 2 & 6 & 5 \end{bmatrix}$

$R_2 \rightarrow \frac{R_2}{2}$

$A \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1/2 \\ 2 & 6 & 5 \end{bmatrix}$

$R_3 \rightarrow R_3 - 2R_1$

$A \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1/2 \\ 0 & 2 & -1 \end{bmatrix}$

$R_3 \rightarrow R_3 - 2R_2$

$A \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore \rho(A) = 2$

Ex 2) find $\rho(A)$

$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 1 & 2 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$

$R_2 \rightarrow R_2 - R_1$

$A \sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 0 & 2 & -1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$

$R_2 \rightarrow R/2$

$A \sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 0 & 1 & -1/2 \\ 1 & 3 & 4 & 5 \end{bmatrix}$

$R_3 \rightarrow R_3 - R_1$ $\quad 3 + \frac{1}{2}$

$A \sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 0 & 1 & -1/2 \\ 0 & 1 & 1 & 3 \end{bmatrix}$

$R_3 \rightarrow R_3 - R_2$

$A \sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 0 & 1 & -1/2 \\ 0 & 1 & 1-0 & 7/2 \end{bmatrix}$

$R_2 \leftrightarrow R_3$

$A \sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 0 & 7/2 \\ 0 & 0 & 1 & -1/2 \end{bmatrix}$

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21) System of non-homogeneous linear equations.

Consider the system of equations

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n &= b_2 \\ \dots \quad \dots \quad \dots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \dots + a_{mn}x_n &= b_m \end{aligned}\right\} \text{①}$$

The system is Known as homogeneous if $b_1 = b_2 = b_m = 0$, otherwise non-homogeneous.

otherwise The system① Can be written in matrix formas

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ - & - & - & - & - \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

$\Rightarrow AX = B$, where. $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ - & - & - & - & - \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ x \\ x_n \end{bmatrix}$, $B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$

The matrix

$$[A:B] = \left[\begin{array}{cccc:c} a_{11} & a_{12} & a_{13} \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & a_{23} \dots & a_{2n} & b_2 \\ - & - & - & - & - \\ a_{m1} & a_{m2} & a_{m3} \dots & a_{mn} & b_m \end{array}\right]$$

is Know as Augmented matrix.

(I) Gauss Elimination Method

Consider the agumented matrix

$$[A:B] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \dots a_{1n} & : b_1 \\ a_{21} & a_{22} & a_{23} \dots a_{2n} & : b_2 \\ - & - & - \quad - & - \\ a_{m1} & a_{m2} & a_{m3} \dots a_{mn} & : b_m \end{bmatrix}$$

~~Step 1~~ Step - $a_{11} \neq 0, a_{22} \neq 0, a_{33} \neq 0 \dots a_{mn} \neq 0$

Step 1 = make the upper triangular matrix for which below the diagonal elements are all zero's

Step 2 = Write down the ~~reverse~~ equation's in reverse order and make the Suitable Substitution, so that we get the value of x.

(II) Gauss Jordan Method.

Consider the agumented matrix

$$[A:B] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \dots a_{1n} & : b_1 \\ a_{21} & a_{22} & a_{23} \dots a_{2n} & : b_2 \\ - & - & - \quad - & - \\ a_{m1} & a_{m2} & a_{m3} \dots a_{mn} & : b_m \end{bmatrix}$$

Step 1 = Use the row transformation, such that

$a_{11} = a_{22} = a_{33} \dots = a_{nn} \neq 0$

Such that $a_{ij} = 0 \ \forall \ i > j$ or $i < j$

then we will get the value of X.

Example 1 :- Solve by Gauss Elimination method.

$x + 2y + z = 3$

$2x + 3y + 3z = 10$

$3x - y + 2z = 13$

→ Solⁿ = The system of eqⁿ's can be written in matrix form as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 13 \end{bmatrix}$$

$\Rightarrow AX = B$

$$\therefore [A:B] = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 2 & 3 & 3 & 10 \\ 3 & -1 & 2 & 13 \end{array}\right]$$

Excellent

Checked 21/09/23

$R_2 \to R_2 - 2R_1$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 3 & -1 & 2 & 13 \end{array}\right]$$

$R_3 \to R_3 - 3R_1$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & -7 & -1 & 4 \end{array}\right]$$

$R_3 \to R_3 - 7R_2$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -8 & -24 \end{array}\right]$$

$\Rightarrow \cancel{-}8z = \cancel{-}24$, $\underline{\underline{z = 3}}$ — ①

Also,

$= -y + z = 4$

$= -y + (3) = 4$

$= -y = 4 - 3$

$\boxed{y = -1}$ — ②

Also, $x + 2y + z = 3$

$\Rightarrow x + 2(-1) + 3 = 3$

$\Rightarrow x - 2 = 0$

$\Rightarrow x = 2$

Hence, $x = 2$, $y = -1$ & $z = 3$ is the required solution.

Ex 2 - Slove by the Gauss Jourdam method

$$x+2y+z=3$$
$$2x+3y+3z=10$$
$$3x-y+2z=13$$

→ Sol^n = the system of equation in the form of

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -8 & -24 \end{array}\right]$$

$R_1 + 2R_2$

x: $2+2(-1)$

y: $2-2$

$1-2(1)$

$1-2+2(-1)$

$2+(-2)$

$1+2(1)$

$3+2(4)$

$3+8$

$R_1 \to R_1 + 2R_2$

$$\left[\begin{array}{ccc:c} 1 & 0 & 3 & 11 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -8 & -24 \end{array}\right]$$

$R_2 \to R_2 + R_3/8$

$$\left[\begin{array}{ccc:c} 1 & 0 & 3 & 11 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -8 & -24 \end{array}\right]$$

$\Rightarrow R_3 \to R_3/-8$

$$\left[\begin{array}{ccc:c} 1 & 0 & 3 & 11 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$R_3 \to R_3/-8$

$$\left[\begin{array}{ccc:c} 1 & 0 & 3 & 11 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$R_2 \to R_2 - R_3$

$$\left[\begin{array}{ccc:c} 1 & 0 & 3 & 11 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$R_1 \to R_1 - 3R_3$

$$\left[\begin{array}{ccc:c} 1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Therefore the value of

$\boxed{x = 2}$, $\boxed{\cancel{y = 1}}$

$\therefore -y = \cancel{1}$

$\therefore \boxed{y = -1}$

Value of $\boxed{z = 3}$

Friday
15/09/23

Janhavi shukla
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Assignment - 1

Q1) Reduce into the Echelon form and find the rank of a matrix $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

→ Solution: $A \sim \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

$R_1 \longleftrightarrow R_2$

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - 3R_1$

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - R_2$

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

0+1

$-2-(-3)$
$-2+3$

$R_4 \longrightarrow R_4 - R_2$

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$R_4 \to R_4 - R_1$

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(A) = 2$

Q2) What is an orthogonal matrix and show that the matrix $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ is an orthogonal matrix.

→ Solution - Orthognal matrix - A square matrix A is said to be orthogonal if $AA^{-1} = I = A'A$.

To verify orthogonality
$AA' = I$

Consider $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore$ It is orthogonal matrix

Q3) Find A^{-1}, where $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

→ Solution - $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ $\therefore |A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 1 & 2 & 2 \end{vmatrix}$

$$|A| = 1(-2-2) - 1(2-1) + 2(2+1)$$
$$= 1(-4) - 1(1) + 2(3)$$
$$= -4 - 1 + 6$$
$$= -5 + 6$$
$$|A| = 1$$
$$\therefore |A| \neq 0$$

Hence A^{-1} exists

Co-factor of elements

1) $1 = (-1)^{1+1} \begin{vmatrix} -1 & 1 \\ 2 & 2 \end{vmatrix} = -2-2 = \underline{\underline{-4}}$

2) $1 = (-1)^{1+2} \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2-1 = \underline{\underline{1}}$

3) $2 = (-1)^{1+3} \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} = 2+1 = \underline{\underline{3}}$

4) $1 = (-1)^{2+1} \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = 2-4 = \underline{\underline{2}}$

5) $-1 = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 2-2 = \underline{\underline{0}}$

6) $1 = (-1)^{2+3} \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2-1 = \underline{\underline{-1}}$

7) $1 = (-1)^{3+1} \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = 1+2 = \underline{\underline{3}}$

8) $2 = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1-2 = \underline{\underline{1}}$

9) $2 = (-1)^{3+3} \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1-1 = \underline{\underline{-2}}$

$\therefore$ Co-factor matrix $A = \begin{bmatrix} -4 & -1 & 3 \\ 2 & 0 & -1 \\ 3 & 1 & -2 \end{bmatrix}$

~~Co-factor~~ m · Adjoint of matrix

$$A = \begin{bmatrix} -4 & 2 & 3 \\ -1 & 0 & 1 \\ 3 & -1 & -2 \end{bmatrix}$$

Inverse of a matrix

i.e. $A^{-1} = \dfrac{\text{adj } A}{|A|}$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} -4 & 2 & 3 \\ -1 & 0 & 1 \\ 3 & -1 & -2 \end{bmatrix}$$

Q4) Solve the system of equations given below by using Gauss Elimination & Gauss Jordan Method.

$$x + y + z = 9$$
$$x - 2y + 3z = 8$$
$$2x + y - z = 3$$

→ (a) Gauss Elimination Method :-

Let $[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 1 & -2 & 3 & 8 \\ 2 & 1 & -1 & 3 \end{array}\right]$ is a agumented matrix.

$R_2 \rightarrow R_2 - R_1$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 2 & 1 & -1 & 3 \end{array}\right]$$

$R_3 \longrightarrow R_3 - 2R_1$

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$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & -1 & -3 & -15 \end{array}\right]$$

$R_3 \rightarrow R_2 - 3R_3$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 11 & 44 \end{array}\right]$$

reverse

Equation $\therefore$ $x+y+z = 9$ — (i)

$-3y+2z = -1$ — (ii)

$11z = 44$

$\boxed{z=4}$

put $z=4$ in equation (ii)

$-3y+2(4) = -1$

$-3y+8 = -1$

$-3y = -1-8$

$+3y = +9$

$\boxed{y=3}$

put $y=3$ and $z=4$ in equation (i)

$\Rightarrow x+3+4 = 9$

$\Rightarrow x+7 = 9$

$\Rightarrow x = 9-7$

$\therefore \boxed{x=2}$

(b) Gauss Jordan Method :-

$$x + y + z = 9$$
$$x - 2y + 3z = 8$$
$$2x + y - z = 3$$

→ The system is in the form of

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 1 & -2 & 3 & 8 \\ 2 & 1 & -1 & 3 \end{array}\right]$$

$$\therefore [A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 11 & 44 \end{array}\right]$$

$$R_1 \rightarrow R_2 + 3R_1$$

$$\therefore [A:B] \sim \left[\begin{array}{ccc:c} 3 & 0 & 5 & 26 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 11 & 44 \end{array}\right]$$

$$R_3 \rightarrow \frac{R_3}{11}$$

$$\therefore [A:B] \sim \left[\begin{array}{ccc:c} 3 & 0 & 5 & 26 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$$R_2 \rightarrow R_2 - 2R_3$$

$$\therefore [A:B] \sim \left[\begin{array}{ccc:c} 3 & 0 & 5 & 26 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$$R_1 \rightarrow R_1 - 5R_3$$

$$\therefore [A:B] \sim \left[\begin{array}{ccc:c} 3 & 0 & 0 & 6 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

The value of

$\therefore \boxed{x = 6}, \boxed{y = -9}$

and $\boxed{z = 4}$

Checked 22/09/23

18/09/23
Monday

Trigonometry and Analytic geometry

1) Angle System :-

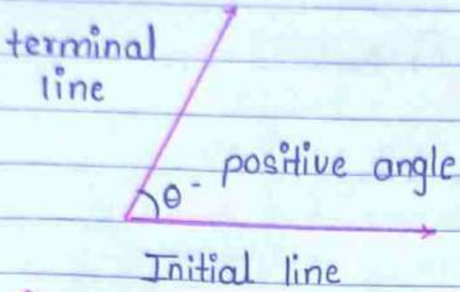

figure-1

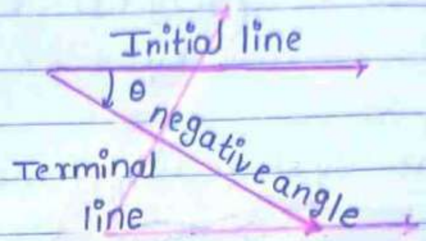

figure-2

1) The rotation of Initial line as shown in figure ① & figure ② is Known as Angle. If it is in anti clockwise then it is Positive angle. otherwise it is negative.

II) If the right angle can be Divided in 90° parts, then we have right angle = 1° = 60', 1' = 60'' which is Known as degree system, similarly if right angle is divided into 100g then we have Right angle. which implise

$$1g = 100'$$
$$1' = 100''$$

* These system is known as grade system.

likewise $\pi^r = 180°$

as $\pi^c = 180°$

$$\boxed{1^c = \frac{180°}{\pi}}$$

$$\boxed{1° = \frac{\pi}{180}}$$

Some standard Angle.

function	0	30°	45°	60°	90°
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞

- Some Identities

- $\sin^2\theta + \cos^2\theta = 1$
- $\sec^2\theta - \tan^2\theta = 1$
- $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

ii) Allied angles and Co-ratios

allied angles are those angles when the Some or Défference of two angles is either zero or multiple of 90°. d.

(1) Example - 30°, 60° are allied angles Because there Sum is 90°.

Wednesday
20/09/23

3) **Allied angles & Co-ratio's** - Allied angles are those angles when the sum or difference of two angles is either zero or multiple of $90°$.

Example = 1) $30°$ & $60°$ are Allied angle. because their sum is $90°$

2) The allied angles for θ are $-\theta$, $90° \pm (-\theta)$, $360° \pm \theta$, etc.

The angle θ is measured in anticlock wise direction as shown below.

S II $90°+\theta$ $180°-\theta$	A I θ $90°-\theta$ $360°+\theta$
T III $180°+\theta$ $270°-\theta$	C IV $270°+\theta$ $360°-\theta$

A ⟶ All trignometric ratio's are +ve.
S ⟶ Sine and Cosine are +ve, rest are -ve.
T ⟶ tan and cot are +ve, rest are -ve.
C ⟶ cos and sec are +ve, rest are -ve

same

Note - (i) At $180° \pm \theta$ and $360° \pm \theta$ trignometric ratio's remains same

(ii) At $270° \pm \theta$ and $90° \pm \theta$ trignometric ratio's change into their Co-ratio's

Change

Thursday
21/09/23

17.

* The co-ratio of $\sin\theta$ is $\cos\theta$
* The co-ratio of $\tan\theta$ is $\cot\theta$
* The co-ratio of $\csc\theta$ is $\sec\theta$

Example - (1) $\sin(90^\circ+\theta) = \cos\theta$

(2) $\sin(270^\circ-\theta) = -\cos\theta$

(3) $\cos(180^\circ-\theta) = -\cos\theta$

(4) $\sin(0-\theta) = -\sin\theta$

(5) $\cos(0-\theta) = \cos\theta$

(6) find the value of $\sin 120^\circ$

$\longrightarrow \sin 120^\circ = \sin(180^\circ-60^\circ)$ $\quad \cancel{\sin(90^\circ+30^\circ)}$

$= \sin 60^\circ$

$= \sqrt{3}/2$

(7) $\cos 210 = \cos(270^\circ-60^\circ)$

$\longrightarrow -\cos(60)^\circ$

$= -1/2$

21/09/23

Exercise 1 - find the values of functions.

1) $\cos 120^\circ$

$= \cos(180^\circ-60^\circ)$

$= -\cos 60^\circ$

$= \frac{-1}{2}$

2) $\tan 210^\circ$

$= \tan(270^\circ-60^\circ)$

$= \cot 60^\circ$

$= \frac{1}{\sqrt{3}}$

3) $\cot 150^\circ$

$= \cot(180^\circ-30^\circ)$

$= -\cot 30^\circ$

$= -\sqrt{3}$

4) $\sec 300^\circ$

$= \sec(360^\circ-60^\circ)$

$= \sec 60^\circ$

$= 2$

Exercise 2:- If $\sin\theta = \frac{-24}{25}$ 'θ' lies in fourth quadrant the find $\cos\theta$ & $\tan\theta$

$\longrightarrow$ Since, we know that,

$$\sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$$

$$\Rightarrow \cos\theta = \sqrt{1-\sin^2\theta}$$

$$\Rightarrow \cos\theta = \sqrt{1-\left(\frac{-24}{25}\right)^2}$$

$$\Rightarrow \cos\theta = \sqrt{\frac{49}{625}}$$

$$\Rightarrow \cos\theta = \pm\frac{7}{25}$$

Since 'θ' lies in fourth quadrant is $\cos\theta = \frac{7}{25}$

Again,

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = \frac{\frac{-24}{25}}{\frac{7}{25}} = \frac{-24}{25} \times \frac{25}{7} = \frac{-24}{7}$$

Exercise 3 :- If 'θ' is located in III quadrant and $\cos\theta = \frac{-3}{5}$, then find $\sin\theta$, $\tan\theta$ & $\cot\theta$.

Solution- We know that

$$\sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$$

$$\Rightarrow \sin\theta = \sqrt{1-\cos^2\theta}$$

$$\Rightarrow \sin\theta = \sqrt{\frac{1}{1} - \frac{9}{25}}$$

$= \sqrt{\frac{25-9}{25}} = \sqrt{\frac{16}{25}}$

$\therefore \sin\theta = \pm\frac{4}{5}$

Since θ lies in III quadrant is $\sin\theta = -\frac{4}{5}$

Again, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{-4}{5}}{\frac{-3}{5}} = \frac{+4}{\cancel{5}} \times \frac{+\cancel{5}}{3} = \frac{4}{3}$

$\therefore \tan\theta = \frac{4}{3}$

again $\cot\theta = \frac{1}{\tan\theta}$

$\therefore \frac{1}{\frac{4}{3}} = \frac{3}{4}$

$\therefore \cot\theta = \frac{3}{4}$

Ans = $\sin\theta = -\frac{4}{5}$, $\tan\theta = \frac{4}{3}$ and $\cot\theta = \frac{3}{4}$.

25/09/23
Monday

4) Compound Angles :- The algebric sum of difference between the angles is known as Compound angles. this angles are used to solve the algebric sum or difference of trignometric function which is state as :-

1) (a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

(b) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

2) (a) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

(b) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

3) (a) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

(b) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

Exercise - find $\sin 15°$ and $\sin 75°$.

Solution - $\sin 15°$ can be written as $\sin(45° - 30°)$

we know,

$\sin(A-B) = \sin A \cos B + \cos A \sin B$

$\Rightarrow \sin 15° = \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$

$\Rightarrow \sin 15° = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$\therefore \sin 15° = \frac{\sqrt{3} - 1}{2\sqrt{2}}$

2) $\sin 75^\circ$

Solution - $\sin 75^\circ$ can be written as

$\sin(~~60~~ 45^\circ + 30^\circ)$

We know,

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\Rightarrow \sin 75^\circ = \sin(45^\circ + 30^\circ)$

$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$\therefore \sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$

Q2) Find the values of $\tan 15^\circ$ and $\sin 105^\circ$

$\longrightarrow$ (1) $\tan 15^\circ$

Soln - $\tan 15^\circ$ can be written as

$\tan(45^\circ - 30^\circ)$

We know,

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$\therefore \tan 15^\circ = \tan(45^\circ - 30^\circ) = \frac{\tan 45^\circ - \tan 30}{1 + \tan 45 \tan 30}$

$\Rightarrow \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}}$

$\Rightarrow \frac{1 - 1/\sqrt{3}}{1 + 1/\sqrt{3}} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

Rationalize

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-1^2}$$

$$= \frac{(\sqrt{3})^2 - 2\sqrt{3}(1) + (1)^2}{3-1}$$

$$= \frac{3-2\sqrt{3}+1}{2} = \frac{4-2\sqrt{3}}{2} = \frac{\cancel{2}(2-\sqrt{3})}{\cancel{2}}$$

$$= 2-\sqrt{3}$$

2) Cot $105°$

→ Cot $105°$ can be written as Cot $(60° + 45°)$

We know,

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\therefore \tan(60° + 45°) = \frac{\tan 60° + \tan 45°}{1 - \tan 60° \cdot \tan 45°}$$

$$= 2 + \sqrt{3}$$

$$\therefore \cot 105° = \frac{1}{\tan 105°}$$

$$\therefore \cot 105° = \frac{1}{2+\sqrt{3}}$$

Exercise 3 - find the value of tan 75° and Cot 15°.

i) tan 75°

→ tan 75° can be written as

tan(45° + 30°)

we know,

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\tan(45° + 30°) = \frac{\tan 45° + \tan 30°}{1 - \tan 45° \cdot \tan 30°}$$

$$= \frac{1 + 1/\sqrt{3}}{1 - 1 \times 1/\sqrt{3}} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}+1}{\cancel{\sqrt{3}}} \times \frac{\cancel{\sqrt{3}}}{\sqrt{3}-1}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Rationalize

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{(\sqrt{3}+1)^2}{(\sqrt{3})^2 - 1^2}$$

$$= \frac{(\sqrt{3})^2 + 2\sqrt{3}(1) + (1)^2}{3-1}$$

$$= \frac{3 + 2\sqrt{3} + 1}{2} = \frac{4 + 2\sqrt{3}}{2} = \frac{\cancel{2}(2+\sqrt{3})}{\cancel{2}}$$

$$= \underline{\underline{2 + \sqrt{3}}}$$

8) Cot 15°

→ Cot 15° can be written as

$\cot(45° - 30°)$

We know

$\tan 15° = 2 - \sqrt{3}$

$\therefore \cot 15° = \dfrac{1}{\tan 15°}$

$\therefore \cot 15° = \dfrac{1}{2-\sqrt{3}}$

Exercise 4):- find the values of Cos 105° and tan 105°.

7) Cos 105°

⟶ Cos 105° can be written as

$\cos(60° + 45°)$

We know,

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\therefore \cos(60° + 45°) = \cos 60° \cdot \cos 45° - \sin 60° \cdot \sin 45°$

$= \left(\dfrac{1}{2}\right)\left(\dfrac{1}{\sqrt{2}}\right) - \left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{1}{\sqrt{2}}\right)$

$= \dfrac{1}{2\sqrt{2}} - \dfrac{\sqrt{3}}{2\sqrt{2}}$

$\Longrightarrow \cos 105° = \dfrac{1-\sqrt{3}}{2\sqrt{2}}$

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2) $\tan 105°$

⟶ $\tan 105°$ can be written as

$\tan(60° + 45°)$

We know,

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\Rightarrow \tan(60° + 45°) = \frac{\tan 60° + \tan 45°}{1 - \tan 60° \cdot \tan 45°}$$

$$= \frac{\sqrt{3} + 1}{1 - \sqrt{3} \cdot 1} = \frac{\sqrt{3}+1}{1-\sqrt{3}} \quad \text{i.e.} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Rationalize.

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{(\sqrt{3}+1)^2}{(\sqrt{3})^2 - 1^2} = \frac{(\sqrt{3})^2 + 2\sqrt{3}(1) + (1)^2}{3-1}$$

$$= \frac{3 + 2\sqrt{3} + 1}{2} = \frac{4 + 2\sqrt{3}}{2} = \frac{\cancel{2}(2+\sqrt{3})}{\cancel{2}}$$

$$\tan 105° = 2 + \sqrt{3}$$

Friday
6/10/23

5) Inverse Trignometric functions :-

if $f: A \rightarrow B$
is a one-one and onto, then
$f^{-1}: B \rightarrow A$ is the inverse function of $f: A \rightarrow B$

• We can Note that : if $y = f(x)$
$\Rightarrow f^{-1}(y) = x$

* There are Six inverse trignometric ratio's who's domain and principle value branch is given as

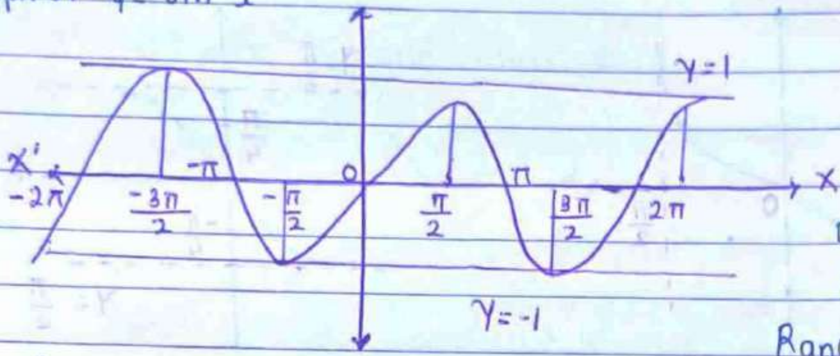
(1) $y = \sin^{-1}x$
Domain = $[-1, 1]$
Principal branch Value = $[-\pi/2, \pi/2]$

(2) $y = \cos^{-1}x$
Domain : $[-1, 1]$
principal branch Value = $[0, \pi]$

(3) $y = \tan^{-1}x$
Domain = $[-\infty, \infty]$
principal branch Value = $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

* Some important graphical ~~wav~~. views of trignometric ratios and inverse trignometric ratios.

(1) Graph of $y = \sin x$



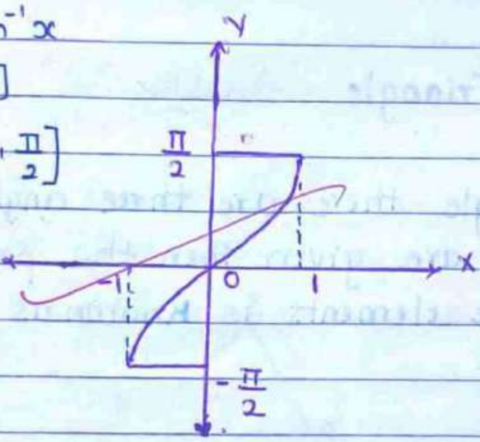
Domain = R = [-∞, ∞]

Range = [-1, 1]

Graph of $y = \sin^{-1} x$

Domain = [-1, 1]

Range = $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



(11) Graph of $y = \cos x$

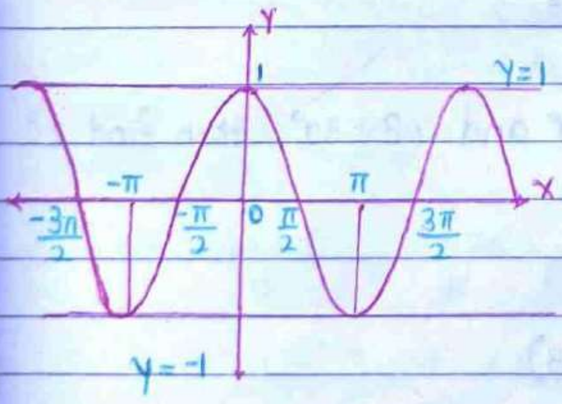
Domain = [-∞, ∞]

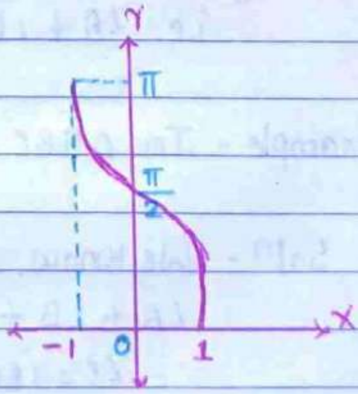
Range = [-1, 1]

Graph of $y = \cos^{-1} x$

Domain = [-1, 1]

Range = [0, π]

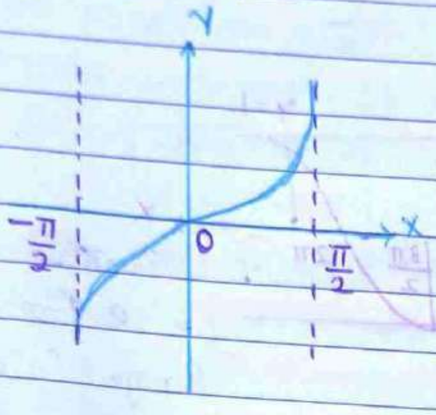




(3) Graph of $y = \tan x$

Domain $= R - \{x : x = (2n+1)\frac{\pi}{2}, n \in Z\}$

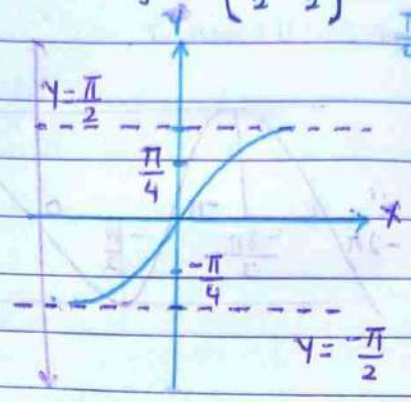
Range $= [-\infty, \infty]$



Graph of $y = \tan^{-1} x$

Domain $= [-\infty, \infty]$

Range $= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



6) Solution of a Triangle

In a triangle there are three angles and three sides if three elements are given then the process of finding the other three elements is known as the solution of a triangle.

(1) No side is given / two angles are given

Sol^n - We know,

the sum of three ~~angles~~ integers angles of triangle is $180°$.

i.e. $\angle A + \angle B + \angle C = 180°$

example - In ΔABC, if $\angle A = 60°$ and $\angle B = 30°$, then find $\angle C$

Sol^n. We know,

$\angle A + \angle B + \angle C = 180°$

$= \angle C = 180° - (\angle A + \angle B)$

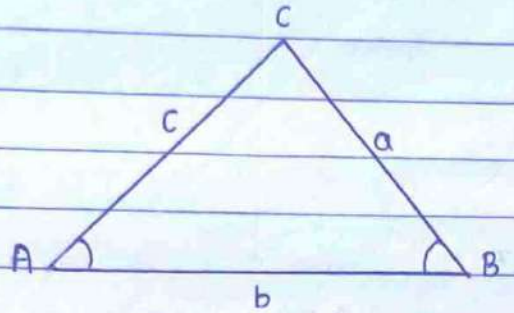
$\angle C = 180° - (60° + 30°)$

$= 180° - 90°$

$= 90°$

(2) One side is given / two angles are given
OR
One angle is given / two sides are given

→ Sol^n - consider a triangle given below.

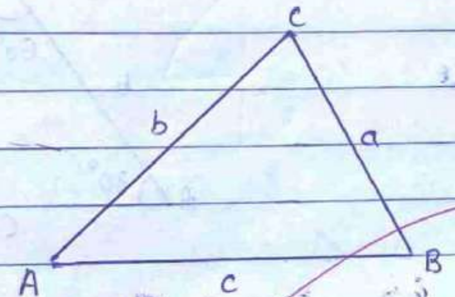


Therefore by sine rule

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Ex]
* find the length of a where angle $A = 40°$ and $\angle C = 70°$ $c = 5$

→ Solution -



∴ by Sin rule

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

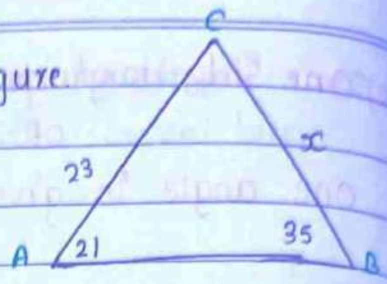
$$\Rightarrow \frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\Rightarrow \frac{\sin 40°}{a} = \frac{\sin 70°}{5}$$

$$a = \frac{\sin 40° \times 5}{\sin 70°}$$

$$a = 3.42$$

Ex2] find the value of 'x' in a given figure



→ We know,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\Rightarrow \frac{\sin A}{a} = \frac{\sin B}{b}$$

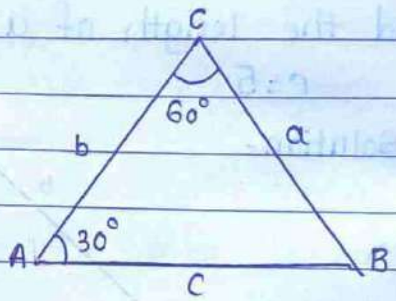
$$\Rightarrow \frac{\sin 21^\circ}{x} = \frac{\sin 35^\circ}{23}$$

$$\Rightarrow x = \frac{\sin 21^\circ}{\sin 35^\circ} \times 23$$

$\Rightarrow$

$$\Rightarrow \text{or } x = \frac{23 \times \sin 21^\circ}{\sin 35^\circ}$$

Ex3] find the length of side 'a' if $\angle A = 30^\circ$ and $\angle C = 60^\circ$ and $c = 10$



→ We know,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

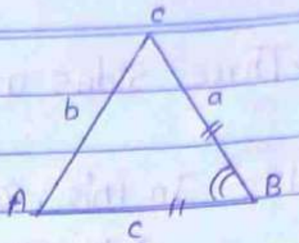
$$\Rightarrow \frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\Rightarrow \frac{\sin 30^\circ}{a} = \frac{\sin 60^\circ}{10}$$

$$\Rightarrow a = \frac{\sin 30^\circ}{\sin 60^\circ} \times 10$$

$$\Rightarrow a = \frac{\frac{1}{\cancel{2}}}{\frac{\sqrt{3}}{\cancel{2}}} \times 10 = \frac{1}{\sqrt{3}} \times 10 = \boxed{a = \frac{10}{\sqrt{3}}}$$

⟨3⟩ Two sides and one included angle



∴ by cosine rule

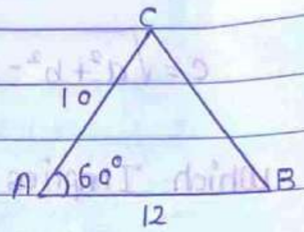
$a = \sqrt{b^2+c^2-2bc\cos A}$

$b = \sqrt{a^2+c^2-2ac\cos B}$

$c = \sqrt{a^2+b^2-2ab\cos C.}$

Ex i] If $b=10$, $c=12$ and $\angle A = 60^\circ$ then find the length of a.

⟶ by cosine rule



$a = \sqrt{b^2+c^2-2bc\cos A}$

$= \sqrt{(10)^2+(12)^2-2(10)(12)\cos 60^\circ}$

$= \sqrt{100+144-240\cos 60^\circ}$

$= \sqrt{4\cancel{0}\cos 60^\circ}$

$= \sqrt{\cancel{4}^{2}\,\frac{1}{\cancel{2}}} = \sqrt{2}$ $\boxed{a=\sqrt{2}}$

12
×2
24
120
4
120
×2
240

iv) Three Sides are given / No angle is given.

Soln :- In this case both sine rule and cosine rule can be used.

i.e. $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

and $a = \sqrt{b^2+c^2-2bc\cos A}$

$b = \sqrt{a^2+c^2-2ac\cos B}$

$c = \sqrt{a^2+b^2-2ab\cos C}$

Which Implies,

$a^2 = b^2+c^2-2bc\cos A$

$\Rightarrow 2bc\cos A = b^2+c^2-a^2$

$\Rightarrow \cos A = \frac{b^2+c^2-a^2}{2bc}$

similary ;

$\cos B = \frac{a^2+c^2-b^2}{2ac}$

$\cos C = \frac{a^2+b^2-c^2}{2ab}$

Ex1] If $a = 10$, $b = 8$ and $c = 7$, then find angle c.

Soln: We know;

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

$$\Rightarrow \cos C = \frac{(10)^2+(8)^2-(7)^2}{2(10)(8)}$$

$$\Rightarrow \cos C = \frac{100+64-49}{160}$$

$$\Rightarrow \cos C = 0.7187$$

$$C = \cos^{-1}(0.7187)$$

Ex2] If $a = b = c = 4$, then find angle A.

→ We know;

$$\cos A = \frac{a^2+b^2-c^2}{2ab}$$

$$= \frac{(4)^2+(4)^2-(4)^2}{2(4)(4)}$$

$$= \frac{32-16}{32} = \frac{16}{32}$$

$$\cos A = \frac{1}{2}$$

$$\therefore \cos A = \cos^{-1}\frac{1}{2}$$

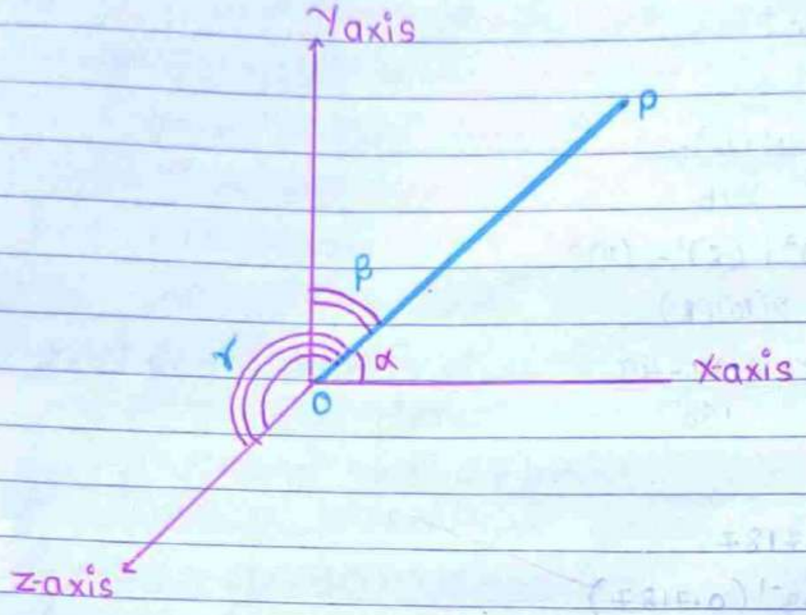
$$\therefore \boxed{A = 60^\circ}$$

Good

Checked 10/10/23

Wednesday
11/10/23

7) Direction Cosines and direction ratios.



Suppose OP be a Vector Such that it makes angle 'α' with x-axis, angle 'β' with Yaxis and angle 'γ' with z-axis, then $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are Known as direction cosines.

These are also denoted by l, m and n respectively and having the relation

$$\boxed{l^2 + m^2 + n^2 = 1}$$

If 'a', 'b' and 'c' are direction ratios, then we have,

(1) $\boxed{\frac{l}{a} = \frac{m}{b} = \frac{n}{c}}$

(2) $l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}}$ $n = \pm \frac{c}{\sqrt{a^2+b^2+c^2}}$

$m = \pm \frac{b}{\sqrt{a^2+b^2+c^2}}$

Note - The numbers proportional to direction cosines of a vector are called direction ratios.

Ex] :- Find the direction cosines of a vector making angles $60°$, $30°$ and $90°$ with x-axis, y-axis and z-axis.

→ Here;

$\alpha = 60°$, $\beta = 30°$, $\gamma = 90°$

Therefore direction cosines are

$\cos\alpha = \cos 60° = \frac{1}{2}$

$\cos\beta = \cos 30° = \frac{\sqrt{3}}{2}$

$\cos\gamma = \cos 90° = 0$

Ex] :- Find the direction cosines of a line having its direction ratios are 2, 1 and -2.

→ Here;

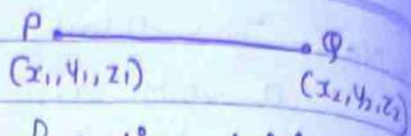
$a = 2$, $b = 1$ and $c = -2$

Therefore,

$$l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}} = \pm \frac{2}{\sqrt{4+1+4}} = \pm \frac{2}{\sqrt{9}} = \pm \frac{2}{3}$$

$$m = \pm \frac{b}{\sqrt{a^2+b^2+c^2}} = \pm \frac{1}{\sqrt{4+1+4}} = \pm \frac{1}{\sqrt{9}} = \pm \frac{1}{3}$$

$$n = \pm \frac{c}{\sqrt{a^2+b^2+c^2}} = \pm \frac{2}{\sqrt{4+1+4}} = \pm \frac{2}{\sqrt{9}} = \pm \frac{2}{9}$$



* Note :- The direction ratios of a line joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are

$$\underset{a}{x_2 - x_1}, \quad \underset{b}{y_2 - y_1}, \quad \underset{c}{z_2 - z_1}$$

And direction cosines are given as :-

$$l = \frac{x_2 - x_1}{|\vec{PQ}|} = \frac{x_2 - x_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

$$m = \frac{y_2 - y_1}{|\vec{PQ}|} = \frac{y_2 - y_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

$$n = \frac{z_2 - z_1}{|\vec{PQ}|} = \frac{z_2 - z_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

12/10/23
Thursday

Janhavi Shukla - BCA - 9944

Assignment - 2

(1)

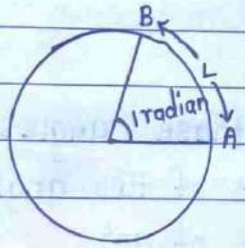
Q1) Explain the degree and radian system with the examples. Convert into radians 15° and 75°.

→ 1) Degree System :- In this system, a right angle is divided into 90 equal parts, called degrees.

1 right angle = 90°
$1^\circ = 60'$
$1' = 60''$

2) Radian System :- In this system, angle is measured in radians. A radian is the angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle.

1 radian is denoted by 1^c.



$L = r\theta$

Relation between Degrees and Radians

$1 \text{ radian} = \left(\frac{180}{\pi}\right)^\circ$

$1 \text{ degree} = \left(\frac{\pi}{180}\right)^c$

Example - (i) Convert into radian measure

Sol^n - 1 degree $= 1^\circ = \left(\frac{\pi}{180}\right)$ radians

$15^\circ = 15 \times \frac{\pi}{180}$ radians

$= \frac{\pi}{12}$ radians

$\therefore \boxed{15^\circ = \left(\frac{\pi}{12}\right)^c}$

(2) Convert 75° into radian.

→ Solⁿ - 1degree = $1^\circ = \left(\frac{\pi}{180}\right)$ radians

$75^\circ = 75 \times \frac{\pi}{180}$ radians

$= 2.4\pi$ radians

$\therefore \boxed{75^\circ = 2.4\pi^c}$

Q2) What are allied angles ? Explain the quadrant System and co-ratios ? Find the Values of sine $(120)^\circ$ and $\cos(210)^\circ$.

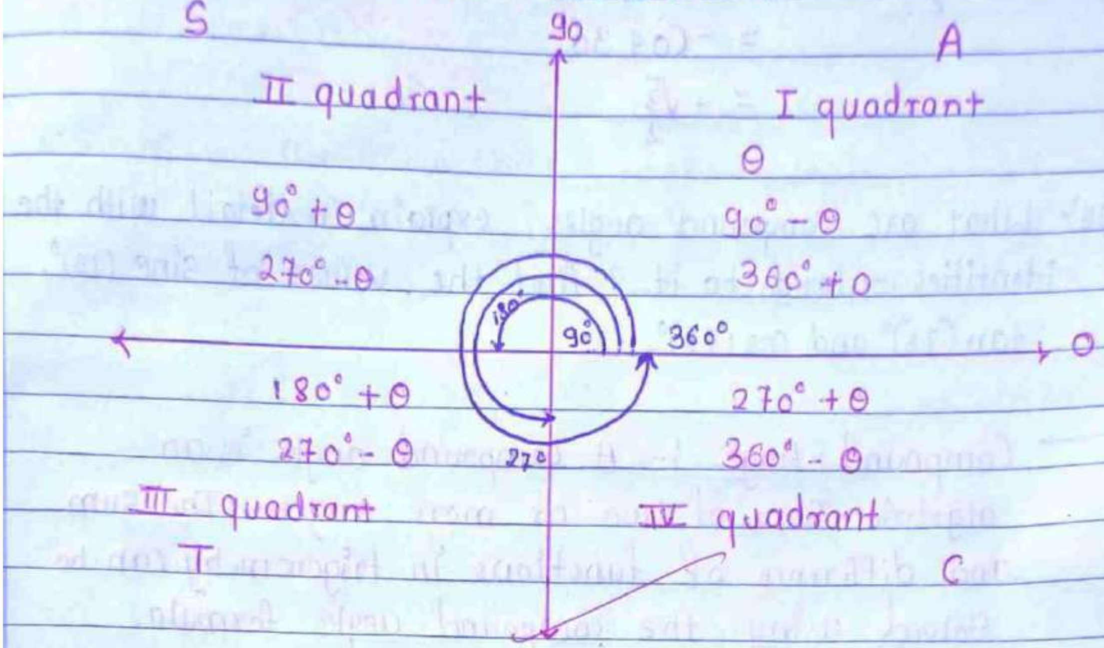
→ Allied Angles :- Allied angles are those angles, when the Sum or difference of two angles is either zero or a multiple of 90°.

Ex. 30° and 60° are allied angle because their Sum is 90°.

Note - (1) The angles $-\theta$, $90^\circ \pm \theta$, $360^\circ \pm \theta$ etc are angles allied to the angle θ, if θ is measured in degrees.

(2) If θ is measured in radians then the angles allied to θ are $-\theta, \frac{\pi}{2} \pm \theta, \pi \pm \theta, 2\pi \pm \theta$ etc.

θ measured in anticlock wise direction.



A ⟶ All trigonometric ratios are +ve

S ⟶ Sine and cosine are +ve, remaining ratios are −ve.

T ⟶ Tan and Cot are +ve, remaing ratios are −ve.

C ⟶ Cos and Sec are +ve, remaing ratios are −ve.

I Note :- Co- ratio's

(1) $\sin\theta \longrightarrow \cos\theta$ (2) $\tan\theta \longrightarrow \cot\theta$

$\cos\theta \longrightarrow \sin\theta$ $\cot\theta \longrightarrow \tan\theta$

(3) $\mathrm{cosec}\,\theta \longrightarrow \sec\theta$

$\sec\theta \longrightarrow \mathrm{cosec}\,\theta$

Example 1) $\sin(120)^\circ$

⟶ consider $\sin 120^\circ = \sin(180^\circ - 60^\circ)$

$= +\sin 60^\circ$

$= \frac{\sqrt{3}}{2}$

2) $\cos 210^\circ = \cos(180^\circ + 30^\circ)$
$= -\cos 30^\circ$
$= -\frac{\sqrt{3}}{2}$

Q3) What are Compound angles, explain in detail with the identities related to it? find the values of sine $(15)^\circ$, $\tan(75)^\circ$ and $\cos(75)^\circ$.

→ Compound Angle :- A compound angle is an algebric sum of two or more angles. The sum and difference of functions in trigonometry can be solved using the compound angle formula. Here we shall deal with functions

like (A+B) and (A-B)

The formulae for trigonometric ratios of compound angles are as follows

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A+B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Example -(i) find the values of sine (15°), tan (75°) and cos (75)°.

(1) $\sin 15°$ = We know that

$\sin 15° = \sin(45° - 30°)$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\therefore \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

(2) $\tan 75°$ = We Know that

$\tan 75° = \tan(45° + 30°)$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$\therefore \tan(45° + 30°) = \frac{\tan 45° + \tan 30°}{1 - \tan 45° \cdot \tan 30°}$

$= \frac{1 + 1/\sqrt{3}}{1 - 1 \times 1/\sqrt{3}} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$

$= \frac{\sqrt{3}+1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}-1}$

$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$

Rationalize

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{(\sqrt{3}+1)^2}{(\sqrt{3})^2 - 1^2}$$

$$= \frac{(\sqrt{3})^2 + 2\sqrt{3}(1) + (1)^2}{3-1}$$

$$= \frac{3+2\sqrt{3}+1}{2} = \frac{4+2\sqrt{3}}{2} = \frac{\cancel{2}(2+\sqrt{3})}{\cancel{2}}$$

$$= \underline{\underline{2+\sqrt{3}}}$$

(3) $\cos(75^\circ)$

→ We know,

$\cos 75^\circ = \cos(45^\circ + 30^\circ)$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(45^\circ + 30^\circ) = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$

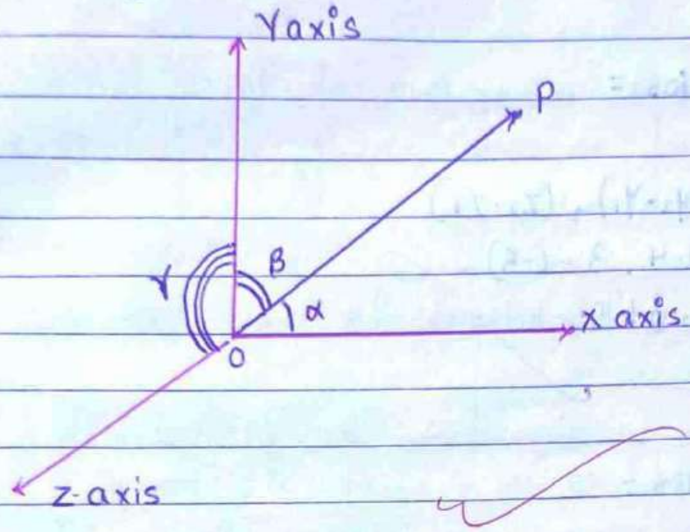
$$\cos(45^\circ + 30^\circ) = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$\cos(75^\circ) = \underline{\underline{\frac{\sqrt{3}-1}{2\sqrt{2}}}}$$

Q4) What are the direction cosine and direction ratios ? Find the direction cosines of the line passing through the points (-2,4,-5) and (1,2,3).

→



Suppose OP be a vector such that it makes angle 'α' with X-axis, angle 'β' with Y-axis and angle 'γ' with Z-axis, then
Cos α , Cos β and cos γ are known as direction Cosines.

These are also denoted by l, m and n respectively and having the relation

$$\boxed{l^2 + m^2 + n^2 = 1}$$

If 'a', 'b' and 'c' are direction ratios, then we have,

(1) $\boxed{\frac{l}{a} = \frac{m}{a} = \frac{n}{c}}$

$\boxed{m = \frac{\pm b}{\sqrt{a^2+b^2+c^2}}}$

(2) $\boxed{l = \frac{\pm a}{\sqrt{a^2+b^2+c^2}}}$

$\boxed{n = \frac{\pm c}{\sqrt{a^2+b^2+c^2}}}$

Solution - Let $P(-2,4,-5)$ and $Q(1,2,3)$

So, $x_1 = -2$, $y_1 = 4$, $z_1 = -5$; $x_2 = 1$, $y_2 = 2$, $z_2 = 3$

Direction ratios =

$$(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)$$
$$= 1-(-2), 2-4, 3-(-5)$$
$$= 1+2, -2, 3+5$$
$$= 3, -2, 8$$

Direction Cosines =

$$\frac{3}{\sqrt{3^2+(-2)^2+8^2}}, \frac{-2}{\sqrt{3^2+(-2)^2+8^2}}, \frac{8}{\sqrt{3^2+(-2)^2+8^2}}$$

$$= \frac{3}{\sqrt{9+4+64}}, \frac{-2}{\sqrt{9+4+64}}, \frac{8}{\sqrt{9+4+64}}$$

$$= \frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$$

Excellent

Checked 13/10/23

~~Friday~~
Monday

Unit-3 Differential Calulus

(i) Limit of Function :-

'l' is said to be limit of function $f(x)$ at $\boxed{x=a}$ if limit x stands for a.

In other word for every $\epsilon > 0$ there exists ($\exists$) $\delta > 0$ such that $|f(x) - l| < \epsilon$, whenever $|x-a| < \delta$

Ex1) find the limit of function $f(x) = x+10$ at $x=5$

$\longrightarrow$

$f(5) = 5+10 = 15$
$f(4.999) = 4.999+10 = 14.999 = 15$
$f(4.9999) = 4.9999 + 10 = 14.9999 = 15$

Now;
$f(5.0001) = 5.0001 + 10 = 15.0001 = 15$
$f(5.00001) = 5.00001 + 10 = 15.00001 = 15$

Also, $\therefore \lim_{x \to 5^-} f(x) = 15$

$\lim_{x \to 5^+} f(x) = 15$

$\lim_{x \to 5} f(x) = 15$

~~Hence~~ Therefore;

$$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x) = \lim_{x \to 5} f(x) = 15$$

Hence, limit of $f(x) = x+10$ at $x=5$ is 15.

Ex2] Find the limit of a function $f(x) = x^2+1$ at $x=2$

$\longrightarrow$ $f(2) = 2^2+1 = 4+1 = 5$

$f(1.99) = (1.99)^2+1 = 4.99 = 5$

$f(1.999) = (1.999)^2+1 = 4.999 = 5$

$f(2.0001) = (2.0001)^2+1 = 5.0001 = 5$

$f(2.00001) = (2.00001)^2+1 = 5.00001 = 5$

Now;

$\underset{2^-}{Lt}\ f(x) = 5$

$\underset{x\to 2^+}{Lt}\ f(x) = 5$

$\underset{x\to 5}{Lt}\ f(x) = 5$

Therefore;

$\underset{x\to 2^-}{Lt}\ f(x) = \underset{x\to 2^+}{Lt}\ f(x) = \underset{x\to 2}{Lt}\ f(x)$

Hence limit of $f(x) = x^2+1$ at $x=2$ is 5.

Ex3] Find the limit of a function $f(x) = 2x+2$ at $x=3$.

$\longrightarrow$ $f(3) = 2(3)+2 = 6+2 = 8$

$f(2.999) = 2(2.999)+2 = 5.999+2 = 8$

$f(2.99) = 2(2.99)+2 = 5.99+2 = 8$

$f(3.001) = 2(3.001)+2 = 6.001+2 = 8$

$f(3.0001) = 2(3.0001)+2 = 6.0001+2 = 8$

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Now;

$$\lim_{x \to 3^-} f(x) = 8$$

$$\lim_{x \to 3^+} f(x) = 8$$

$$\lim_{x \to 3} f(x) = 8$$

Therefore;

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = \lim_{x \to 3} f(x)$$

Hence limit of $f(x) = 2x+2$ at $x = 3$ is 8.

23/10/23
Monday

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Question Bank Solutions.

1) The Co-ratio of sec θ is Cosec θ

2) $45° = \frac{\pi}{4}$ radians.

3) The reciprocal ratio of tanθ is Cotθ.

4) The value of sin(120) is $\frac{\sqrt{3}}{2}$

5) Sin(A+B) = SinA CosB + CosA SinB.

6) Find transpose of the matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ 12 & 41 & -6 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 12 \\ -2 & 41 \\ 1 & -6 \end{bmatrix}$$

-1-0

7) If $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ then its determinant is $\underline{\underline{-1}}$

8) Write an example of a skew-symmetric matrix.

$$= \begin{bmatrix} 0 & 11 & -2 \\ -11 & 0 & 4 \\ 2 & -4 & 0 \end{bmatrix}$$

9) Define the term "orthogonal matrix".

→ If $A'A = AA' = I$, then matrix A is called an orthogonal matrix.

10) If $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ then order of A.B is

$A \cdot B = \begin{bmatrix} 1+0 & 1+0 \\ 2+0 & 2+(1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ Ans = 2×2

11) 3 radians = $\frac{540}{\pi}$ $^{\circ}$

12) $15^{\circ} = \frac{\pi}{12}^{c}$

13) The reciprocal ratio of $\tan\theta$ is $\cot\theta$

14) The value of $\sin(270-\theta)$ is $-\cos\theta$

$= -\cos$

15) $\cos(A+B) = \cos A \cos B - \sin A \sin B$.

16) The order of the matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 6 \end{bmatrix}$ is $= \underline{2 \times 3}$.

17) If $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ then its transpose is $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

18) Write an example of a symmetric matrix

$A' = A$

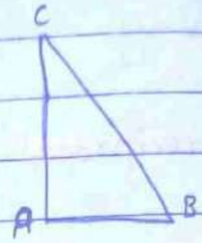
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 6 \\ 3 & 6 & 9 \end{bmatrix} = A' = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

19) Define the term "singular matrix".

The determinant of a matrix is a real number and is denoted as $|A|$. if $|A| = 0$, then A is said to be a singular matrix.

20) If $A = \begin{bmatrix} 3 & 4 \\ 7 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then $A+B$ is $\begin{bmatrix} 4 & 6 \\ 10 & 12 \end{bmatrix}$

21) Find the remaining trigonometric ratios of the angle A, if $\tan A = \frac{4}{3}$



→ $\tan A = \frac{4}{3}$

$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{O}{a} = \frac{4}{3}$

Here, Oppo $O = 4$ and $a = 3$, $h = ?$

By pythagoras theorem,

$h^2 = O^2 + a^2$

$\Rightarrow h = \sqrt{O^2 + a^2}$

$\Rightarrow h = \sqrt{(4)^2 + (3)^2}$

$\Rightarrow h = \sqrt{16+9} = \sqrt{25} = 5$

$\therefore h = 5$

Now,

$\sin A = \frac{O}{h} = \frac{4}{5}$

$\cos A = \frac{a}{h} = \frac{3}{5}$

$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{5}{4}$

$\sec A = \frac{1}{\cos A} = \frac{5}{3}$

$\cot A = \frac{1}{\tan A} = \frac{3}{4}$

Hence, other trigonometric ratios are:

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$\sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$, $\operatorname{cosec} A = \frac{5}{4}$, $\sec A = \frac{5}{3}$

$\cot A = \frac{3}{4}$

22) If $\sin A = \frac{3}{5}$ find the remaining trigonometric ratios of the angle A.

→ $\sin A = \frac{3}{5}$

We know, $\sin A = \frac{\text{Opposite}}{\text{hypo}} = \frac{o}{h} = \frac{3}{5}$

Here, $o = 3$ and $h = 5$, $a = ?$

By pythagoras theorem

$h^2 = o^2 + a^2$

$\Rightarrow h^2 - o^2 = a^2$

$\Rightarrow a = \sqrt{h^2 - o^2}$

$\Rightarrow a = \sqrt{(5)^2 - (3)^2}$

$\Rightarrow a = \sqrt{25 - 9}$

$\Rightarrow a = \sqrt{16}$

$\therefore \boxed{a = 4}$

Now,

$\cos A = \frac{a}{h} = \frac{4}{5}$

$\tan A = \frac{o}{a} = \frac{3}{4}$

$\cot A = \frac{1}{\tan A} = \frac{4}{3}$

$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{5}{3}$

$\sec A = \frac{1}{\cos A} = \frac{5}{4}$

Ans: $\cos A = \frac{4}{5}$

$\tan A = \frac{3}{4}$

$\cot A = \frac{4}{3}$

$\operatorname{cosec} A = \frac{5}{3}$

$\sec A = \frac{5}{4}$

Q23) Find the determinant of the matrix $B = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$\rightarrow$
$$= 2(6-2)-2(2-1)+1(2-3)$$
$$= 2(4)-2(1)+1(-1)$$
$$= 8-2-1$$
$$= \underline{5}$$

Q24) What is an orthogonal matrix and show that the matrix $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ is an orthogonal matrix.

$\rightarrow$ If $AA' = A'A = I$, then matrix A is called as orthogonal matrix

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AA' = I$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore AA' = I$$

Hence, ~~ar~~ matrix $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ is an orthogonal matrix.

6

Q25) Find the determinant of the matrix $P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 3 & 11 \\ 1 & 0 & -2 \end{bmatrix}$

→ $P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 3 & 11 \\ 1 & 0 & -2 \end{bmatrix}$

$= 1(-6-0) - 0 + 1(0-3)$

$= 1(-6) + 1(-3)$

$= -6 - 3$

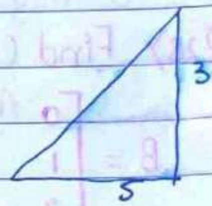
$= \underline{\underline{-9}}$

Q26) If $\tan C = \frac{3}{5}$, find the remaining trigonometric ratios of the angle C.

→ $\tan C = \frac{3}{5}$

We know,

$\tan C = \frac{\text{Oppo}}{\text{adja}} = \frac{o}{a} = \frac{3}{5}$



Here $o = 3$ and $a = 5$, $h = ?$

By Pythagoras theorem

$\Rightarrow h^2 = o^2 + a^2$

$\Rightarrow h = \sqrt{o^2 + a^2}$

$\Rightarrow h = \sqrt{(3)^2 + (5)^2}$

$\Rightarrow h = \sqrt{9 + 25}$

$\Rightarrow h = \sqrt{34}$

$\boxed{h = \sqrt{34}}$

Now,

$\sin C = \frac{3}{\sqrt{34}}$

$\operatorname{cosec} C = \frac{1}{\sin C} = \frac{\sqrt{34}}{3}$

$\cos C = \frac{5}{\sqrt{34}}$

$\sec C = \frac{1}{\cos C} = \frac{\sqrt{34}}{5}$

$\cot C = \frac{1}{\tan C} = \frac{5}{3}$

$-1+2$

$-1+4$

Q27) Find the Value of A^2, where $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & 4 & 2 \end{bmatrix}$

$\rightarrow A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & 4 & 2 \end{bmatrix}$

$$A^2 = A \cdot A \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 1+0+3 & 2+2+4 & 1-2+2 \\ 0+0-3 & 0+1-4 & 0-1-2 \\ 3+0+6 & 6+4+8 & 3-4+4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4 & 8 & 1 \\ -3 & -3 & -3 \\ 9 & 18 & 3 \end{bmatrix}$$

Q28) Find $(A+B)$ and $(A \cdot B)$, where $A = \begin{bmatrix} 1 & 1 & 3 \\ -1 & 0 & 2 \\ -2 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 4 & 1 \\ 2 & 0 & 1 \end{bmatrix}$

$$\rightarrow (A+B) = \begin{bmatrix} 1 & 1 & 3 \\ -1 & 0 & 2 \\ -2 & 1 & 5 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 3 \\ 1 & 4 & 1 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 4 & 3 \\ 0 & 1 & 6 \end{bmatrix}$$

$$(A \cdot B) = \begin{bmatrix} 1 & 1 & 3 \\ -1 & 0 & 2 \\ -2 & 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 & 3 \\ 1 & 4 & 1 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0+1+6 & -1+4+0 & 3+1+3 \\ 0+0+4 & 1+0+0 & -3+0+2 \\ 0+1+10 & 2+4+0 & -6+1+5 \end{bmatrix}$$

$$(A \cdot B) = \begin{bmatrix} 7 & 3 & 7 \\ 4 & 1 & -1 \\ 11 & 6 & 0 \end{bmatrix}$$

Q29) Solve the system of equations given below by using Gauss Jordan Method. (a) $x+y+z=9$
(b) $x-2y+3z=8$
(c) $2x+y-z=3$

$\rightarrow$ $AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ 3 \end{bmatrix}$$

Let $[A:B]$ $\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 1 & -2 & 3 & 8 \\ 2 & 1 & -1 & 3 \end{array}\right]$ is a agumented matrix

$R_2 \rightarrow R_2 - R_1$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 2 & 1 & -1 & 3 \end{array}\right]$$

$R_3 \rightarrow R_3 - 2R_1$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & -1 & -3 & -15 \end{array}\right]$$

$R_3 \rightarrow R_2 - 3R_3$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 11 & 44 \end{array}\right]$$

$R_3 \rightarrow \frac{R_3}{11}$

$\cancel{3}$ $\cancel{0}$ $\cancel{5}$

9

$R_2 \to R_2 - 2R_3$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$R_2 \to R_2 - 2R_3$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$R_1 \to R_1 - R_3$

$R_1 \to 3R_1 + R_2$

$$\left[\begin{array}{ccc:c} 1 & 1 & 0 & 5 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$R_1 \to 3R_1 + R_2$

$$\left[\begin{array}{ccc:c} 3 & 0 & 0 & 6 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$\therefore$ $\boxed{x=6}$, $\boxed{y=-9}$ and $\boxed{z=4}$

30) Find the rank of A, by reducing into Echelon form,

where $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

$\longrightarrow$ $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 2 & 6 & 5 \end{bmatrix}$$

$R_2 \longrightarrow \frac{R_2}{2}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1/2 \\ 2 & 6 & 5 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1/2 \\ 0 & 2 & -1 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - 2R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence, a rank of a matrix A is $\underline{2}$.

Q31) Reduce into the Echelon form and find the rank of a matrix $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

$\longrightarrow$ $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

$R_1 \longleftrightarrow R_2$

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1$

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$R_4 \to R_4 - R_2$

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$R_4 \to R_4 - R_1$

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(A) = 2$

Q32) Solve the system of equations given below by using Gauss Elimination.

a. $x + y + z = 9$

b. $x - 2y + 3z = 8$

c. $2x + y - z = 3$

→ $AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ 3 \end{bmatrix}$$

Let $[A:B]$ $\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 1 & -2 & 3 & 8 \\ 2 & 1 & -1 & 3 \end{array}\right]$ is a agumented matrix

$R_2 \rightarrow R_2 - R_1$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 2 & 1 & -1 & 3 \end{array}\right]$$

$R_3 \rightarrow R_3 - 2R_1$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & -1 & -3 & -15 \end{array}\right]$$

$R_3 \rightarrow R_2 - 3R_3$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 11 & 44 \end{array}\right]$$

reverse equation :- $x + y + z = 9$ — (1)

$-3y + 2z = -1 \quad \text{— (ii)}$

$11z = 44 \quad \text{— (iii)}$

$\boxed{z = 4}$

put z=4 in equation (ii)

$-3y + 2(4) = -1$

$-3y + 8 = -1$

$-3y = -1 - 8$

$\cancel{-}3y = \cancel{-}9$

$\boxed{y = 3}$

put y=3 in ~~eqn~~ (ii) and z=4 in eqn (i)

$x + 3 + 4 = 9$

$x + 7 = 9$

$x = 9 - 7$

$\therefore \boxed{x = 2}$

Hence, the value of x, y and z are 2, 3 and 4 respectively.

Excellent

Peer 25/10/23

Notes

provided

by

Vaibhavi Jadhav