28/11/23
Monday

Unit-3 = differential Calculus

Continuation

* **Continuity** :- The $f(x)$ is said to be continue at point $x = c$ if $\lim_{x \to c} f(x) = f(c)$

In other words, for any $\epsilon > 0$ there exists $\delta > 0$, Such that

$$|f(x) - f(c)| < \epsilon$$

whenever $|x-c| < \delta$

The function is said to be continuous in an interval / [a,b] if it is continuous at every point of interval.

example 1 - Examine the continuity of a function $f(x) = 2x^2-1$ at $x = 3$

Solution - $f(3) = 2(3)^2 - 1 = (2)(9) - 1 = 18 - 1 = \underline{17}$

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} (2x^2-1) = 2(3)^2 - 1 = \underline{17}$$

example 2 - Examine the continuity of $f(x) = x^3 - 2x + 1$ at $x = 2$

$\rightarrow$ $f(2) = (2)^3 - 2(2) + 1 = 8 - 4 + 1 = 4 + 1 = \underline{5}$

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} (x^3 - 2x + 1) = (2)^3 - 2(2) + 1 = \underline{5}$$

Therefore;

$$\lim_{x \to 2} (x^3 - 2x + 1) = f(2)$$

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Note - If $f(x)$ and $g(x)$ are two continuous function at point $x=c$
then, $f(x)+g(x)$, $f(x)-g(x)$, $f(x).g(x)$ and $\frac{f(x)}{g(x)}, g(x) \neq 0$ are continuous function at $x=c$

(3) Differentiability of a Function

If $f(x)$ is a function such that $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ exists
then this is known as differentiability of $f(x)$ at $x=a$

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Some Important derivations

1) $\frac{dx^n}{dx} = nx^{n-1}$

2) $\frac{dc}{dx} = 0$

3) $\frac{d}{dx} = \frac{1}{|x|}$

4) $\frac{d}{dx} e^{ax} = e^{ax}.a$

5) $\frac{d}{dx} \sin x = \cos x$

6) $\frac{d}{dx} \cos x = -\sin x$

7) $\frac{d}{dx} \tan x = \sec^2 x$

8) $\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$

9) $\frac{d}{dx} \sec x = \sec\tan x$

10) $\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$

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11) $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{d}{dx}v$ $[u-v] = \frac{du}{dx} - \frac{dv}{dx}$

12) $\frac{d}{dx}(uv) = u\frac{d}{dx}v + v\frac{d}{dx}u$

13) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}u - v\frac{d}{dx}v}{v^2}$

Exercise :- Find the derivative of

(1) $f(x) = 2x^2 + 4x + 1$

→ Here,

$f(x) = 2x^2 + 4x + 1$

diff w.r.t 'x' we get

$\frac{d}{dx}f(x) = \frac{d}{dx}(2x^2 + 4x + 1)$

$= \frac{d}{dx}2x^2 + \frac{d}{dx}4x + \frac{d}{dx}1$

$= 2\frac{dx^2}{dx} + 4\frac{d}{dx}x + 0$

$= 2(2x^1) + 4(1) + 0$

$= \underline{4x + 4}$

$= \cancel{8x}$

Note = $\frac{d}{dx}(x)$ - How ?

$$\frac{d}{dx}x^1 = 1\cdot x^{1-1}$$

$$= 1\cdot x^0$$

$$= 1\cdot 1$$

$$\boxed{= 1}$$

Ex2] Find $f'(x)$, where $f(x) = \sin^2 x + 2\cos x$.

→ Solution - Here,

$$f(x) = \sin^2 x + 2\cos x$$

diff w.r.t 'x' we get

$$f'(x) = \frac{d}{dx}\sin^2 x + \frac{d}{dx} 2\cos x$$

$$= 2\sin x^{2-1}\cdot \cos x + 2(-\sin x)$$

$$= 2\sin x\cdot \cos x - 2\sin x$$

Ex3] If $f(x) = \sqrt{3} + 4x^3 + 1$ then Find $F'(x)$.

→ Here $F(x) = \sqrt{3} + 4x^3 + 1$

diff w.r.t 'x' we get

$$f(x) = \frac{1}{2}(3)^{1/2 - 1} + 4\cdot 3\cdot x^{3-1} + 0$$

$$= \frac{1}{2} 3^{-1/2} + 12x^2 + 0$$

$$= \frac{1}{2\sqrt{3}} + 12x^2$$

Ex 3] $f(x) = 2e^x - 4\log x$

→ Given function $f(x) = 2e^x - 4\log x$

differentiate w r to x, we get

$$\frac{df}{dx} = f'(x) = \frac{d}{dx}[2e^x - 4\log x]$$

We know that $\frac{d}{dx}[u-v] = \frac{du}{dx} - \frac{dv}{dx}$

$$\therefore f'(x) = \frac{d}{dx}[2e^x] - \frac{d}{dx}[4\log x]$$

We know that $\frac{d}{dx}[cu] = c\frac{du}{dx}$

$$\therefore f'(x) = 2\frac{d}{dx}[e^x] - 4\frac{d}{dx}[\log x]$$

We know that $\frac{d}{dx}[e^x] = e^x$ & $\frac{d}{dx}[\log x] = \frac{1}{x}$

$$\therefore f'(x) = 2(e^x) - 4(\log x)$$

Hence $f'(x) = 3e^x - 4\log x$

Ex 4] $f(x) = 2\sin x - 3\cos x$

→ Given function $f(x) = 2\sin x - 3\cos x$

diff w.r.to x, we get

$$\frac{df}{dx} = f'(x) = \frac{d}{dx}[2\sin x - 3\cos x]$$

We know that $\frac{d}{dx}[u-v] = \frac{du}{dx} - \frac{dv}{dx}$

$$\therefore f'(x) = \frac{d}{dx}[2\sin x] - \frac{d}{dx}[3\cos x]$$

We know that $\frac{d}{dx}[cu] = c\frac{du}{dx}$

$$\therefore f'(x) = 2\frac{d}{dx}[\sin x] - 3\frac{d}{dx}[\cos x]$$

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We know that $\frac{d}{dx}[\sin x] = \cos x$ & $\frac{d}{dx}[\cos x] = -\sin x$

$\therefore f'(x) = 2[\cos x] - 3[-\sin x]$

Hence $f'(x) = 2\cos x + 3\sin x$

5) $f(x) = 3\tan x - 4x$

→ Given function $f(x) = 3\tan x - 4x$

diff w.r.to x, we get

$\frac{df}{dx} = f'(x) = \frac{d}{dx}[3\tan x - 4x]$

We know that $\frac{d}{dx}[u-v] = \frac{du}{dx} - \frac{dv}{dx}$

$\therefore f(x) = \frac{d}{dx}[3\tan x] - \frac{d}{dx}[4x]$

We know that $\frac{d}{dx}[cu] = c\frac{du}{dx}$

$\therefore f'(x) = 3\frac{d}{dx}[\tan x] - 4\frac{d}{dx}[x^1]$

We know that $\frac{d}{dx}[\tan x] = \sec^2 x$ & $\frac{d}{dx}[x^n] = nx^{n-1}$

~~6) $f(x) = 5x^2 + 2\cot x - 3\log x$~~

$\therefore f'(x) = 3[\sec^2 x] - 4[1 * x^{1-1}] = 3\sec^2 x - 4[1]$

Hence $f'(x) = 3\sec^2 x - 4$

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6) $f(x) = 5x^2 + 2\cot x - 3\log x$

$\rightarrow$ Given function $f(x) = 5x^2 + 2\cot x - 3\log x$

Diff. w.r.to x, we get

$$\frac{df}{dx} = f'(x) = \frac{d}{dx}\left[5x^2 + 2\cot x - 3\log x\right]$$

We know that $\frac{d}{dx}[u+v-z] = \frac{du}{dx} + \frac{dv}{dx} - \frac{dz}{dx}$

$\therefore f'(x) = \frac{d}{dx}[5x^2] + \frac{d}{dx}[2\cot x] - \frac{d}{dx}[3\log x]$

We know that $\frac{d}{dx}[cu] = c\frac{du}{dx}$

$\therefore f'(x) = 5\frac{d}{dx}[x^2] + 2\frac{d}{dx}[\cot x] - 3\frac{d}{dx}[\log x]$

We know that $\frac{d}{dx}[x^n] = nx^{n-1}$, $\frac{d}{dx}[\cot x] = -\operatorname{cosec}^2 x$ &

$\frac{d}{dx}[\log x] = \frac{1}{x}$

$\therefore f'(x) = 5\cancel{\frac{d}{dx}}\left[2 * x^{2-1}\right] + 2\left[-\operatorname{cosec}^2 x\right] - 3\left[\frac{1}{x}\right]$

$= 5[2x^1] - 2\operatorname{cosec}^2 x - \frac{3}{x}$

Hence $f'(x) = 10x - 2\operatorname{cosec}^2 x - \frac{3}{x}$

7) $f(x) = 3\operatorname{cosec} x + \sec x$

$\longrightarrow$ Given function $f(x) = 3\operatorname{cosec} x + \sec x$
diff w.r.to x, we get

$$\frac{df}{dx} = f'(x) = \frac{d}{dx}[3\operatorname{cosec} x + \sec x]$$

We know that $\frac{d}{dx}[u+v] = \frac{du}{dx} + \frac{dv}{dx}$

$$\therefore f'(x) = \frac{d}{dx}[2\operatorname{cosec} x] + \frac{d}{dx}[\sec x]$$

We know that $\frac{d}{dx}[cu] = c\frac{du}{dx}$

$$\therefore f'(x) = 2\frac{d}{dx}[\operatorname{cosec} x] + \frac{d}{dx}[\sec x]$$

We know that $\frac{d}{dx}[\operatorname{cosec} x] = -\operatorname{cosec}\cot x$ & $\frac{d}{dx}[\sec x] = \sec x \tan x$

$$\therefore f'(x) = 2[-\operatorname{cosec} x \cot x] + [\sec x \tan x]$$

Hence $f'(x) = -2\operatorname{cosec} x \cot x + \sec x \tan x$

8] $f(x) = 2x^2 \sin x$

$\longrightarrow$ Given function $f(x) = 2x^2 \sin x$
diff w.r.to x, we get

$$\frac{df}{dx} = f'(x) = \frac{d}{dx}[(2x^2)(\sin x)]$$

We know that $\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}$ (product rule)

$$\therefore f'(x) = (2x^2)\frac{d}{dx}[\sin x] + (\sin x)\frac{d}{dx}[2x^2]$$

We know that $\frac{d}{dx}[\sin x] = \cos x$ & $\frac{d}{dx}[x^n] = nx^{n-1}$

$\therefore f'(x) = (2x^2)(\cos x) + (\sin x)[2(2 * x^{2-1})]$

Hence $f'(x) = 2x^2 \cos x + \sin x(4x) = 2x^2 \cos x + 4x \sin x$

9) $f(x) = \frac{\cos x}{x^2}$

→ Given function $f(x) = \frac{\cos x}{x^2}$

diff w.r.t. to x, we get

$\frac{df}{dx} = f'(x) = \frac{d}{dx}\left[\frac{\cos x}{x^2}\right]$

We know that $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ (Quotient rule)

$\therefore f'(x) = \frac{x^2 \frac{d}{dx}[\cos x] - \cos x \frac{d}{dx}[x^2]}{(x^2)^2}$

We know that $\frac{d}{dx}[\cos x] = -\sin x$ & $\frac{d}{dx}[x^n] = nx^{n-1}$

$\therefore f'(x) = \frac{x^2[-\sin x] - \cos x[2 * x^{2-1}]}{x^4}$

Hence $f'(x) = \frac{-x^2 \sin x - 2x\cos x}{x^4}$

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1/12/23
Friday

4) Chain Rule :- If $y = f(u)$ is a function in u and u is itself a function t i.e. $u = f(t)$. then the process of finding derivative of y w.r.t t

i.e. $\frac{dy}{dt}$ is known as a chain rule.

i.e. $\boxed{\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt}}$

Q1) Find the derivative by chain rule of

$f(x) = (4x^2+3)^3$

→ Solution - Here, $f(x) = (4x^2+3)^3$

let $y = (4x^2+3)^3 \Rightarrow y = u^3$
$u = 4x^2+3$

∴ By chain rule, we have

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \quad \text{— (1)}$$

$$\frac{dy}{du} = 3u^2 \text{ and } \frac{du}{dx} = 8x$$

Therefore from (1); we have;

$$\frac{dy}{dx} = 3u^2 \cdot 8x$$

$$= 24(4x^2+3)^2 \cdot x$$

$$= 24x(4x^2+3)^2$$

[Ex2] find $\frac{dy}{dx}$ by using chain rule, where; $y = (2x-3)^4$

→ Solution: Here; ~~f(y)=~~ $y = (2x-3)^4$

let $y = u^4 \Rightarrow (2x-3)^4$

$u = 2x-3$

∴ By chain rule, we have

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \quad -①$$

$$\frac{dy}{dx} = 4u^3 \text{ and } \frac{du}{dx} = 2$$

Therefore from ① ; we have ;

$$\frac{dy}{dx} = 4u^3 \cdot 2$$

$$= 8(2x-3)^3$$

( ) → keandar
[ ] → lekar

5) Rolle's Theorem : It state's that if $f(x)$ is continuous in $[a,b]$, $f(x)$ is differentiable in (a,b); $f(a) = f(b)$ Then $c \in (a,b)$ and $f'(c)=$ such that $f'(c) = 0$

Ex = Verify Rolle's theorem

$f(x) = x^2+2 \quad (-2,2)$

Solution - Clearly (i) $f(x) = x^2+2$ is continuous in $[a,b]$ $[-2,2]$

(ii) $f(x) = x^2+2$ is differentiable in $(-2,2)$

(iii) Now, $f(-2) = f(2)$

$f(-2) = (-2)^2+2 = 4+2 = 6$

$f(2) = (2)^2+2 = 4+2 = 6$

Now ;

$f(x) = x^2+2$

$f'(x) = 2x$

$f'(c) = 2c$

$\Rightarrow f'(c) = 0$

$\Rightarrow 2c = 0$

$\Rightarrow c = 0 \in (-2,2)$

Hence Rolle's theorem verified.

Ex2] Verify Rolle's theorem x^2-6x+5 in $(0,6)$

→ clearly (i) $f(x) = x^2-6x+5$ is continuous in $[0,6]$

(ii) $f(x) = x^2-6x+5$ is differentiable in $(0,6)$

(iii) Now, $f(0) = f(6)$

$f(0) = (0)^2-6(0)+5 = 0-0+5 = 5$

$f(6) = (6)^2-6(6)+5 = 36-36+5 = 5$

Now ;

$f(x) = x^2-6x+5$

$f'(x) = 2x-6$

$f'(c) = 2c-6$

$\Rightarrow f'(c) = 0$

$\Rightarrow 2c - 6 = 0$

$\Rightarrow \cancel{c \neq 0 \in [0,6)}$

~~Hence Rolle's~~

$\Rightarrow 2c = 0 + 6$

$\Rightarrow 2c = 6 \quad \therefore c = 6/2$

$\boxed{c = 3}$

$\therefore c = 3 \in (0,6)$

Ex3] verify Rolle's theorem $x^2 + 2x - 8$ in $(-4,2)$

$\longrightarrow$ Clearly (i) $f(x) = x^2 + 2x - 8$ is continuous in $[-4,2]$

(ii) $f(x) = x^2 + 2x - 8$ is differentiable in $(-4,2)$

(iii) Now, $f(-4) = f(2)$

$f(-4) = (-4)^2 + 2(-4) - 8$

$= 16 - 8 - 8 = 8 - 8 = 0$

$f(2) = (2)^2 + 2(2) - 8 = 4 + 4 - 8 = 0$

Now;

$f(x) = x^2 + 2x - 8$

$f'(x) = 2x + 2$

$f'(c) = 2c + 2$

$f'(c) = 0$

$2c + 2 = 0$

$2c = 0 - 2 \Rightarrow c = \frac{-\cancel{2}}{\cancel{2}}, \quad \boxed{c = -1}$

$\therefore c = -1 \in (-4,2)$

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4/12/23
Monday

* Lagrange's Mean Value Theorem (or Mean-Value Theorem)

Statement : If $f(x)$ is conditinuous on $[a,b]$ and differentiable in (a,b), then there exist at least one value c in (a,b) such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

1) Verify Lagrange's mean value theorem for the function $f(x) = x^2$ in $(2,4)$.

→ The Mean Value Theorm is given by

If $f(x)$ is continuous on $[a,b]$ and differentiable in (a,b), then there exist at least one value c in (a,b) such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

Given $f(x) = x^2$ ----- ① & $a=2, b=4$

Clearly given function $f(x) = x^2$ which is being a polynomial, is continuous $[2,4]$ and differentiable in $(2,4)$

$$f(a) = f(2) = (2)^2 = 4$$
$$f(b) = f(4) = (4)^2 = 16$$

~~Clear~~ Differentiate (1) with respect to x, we get

$$f'(x) = 2x \Rightarrow f'(c) = 2c \quad \therefore$$

Let $f'(c) = \dfrac{f(b)-f(a)}{b-a} \Rightarrow 2c = \dfrac{f(4)-f(2)}{4-2} \Rightarrow 2c = \dfrac{16-4}{2}$

$\Rightarrow 2c = 6 \Rightarrow c = 3$

$\therefore c = 3 \in (2,4)$ such that $f'(3) = 6 = \dfrac{f(4)-f(2)}{4-2}$

Hence Lagrange's means value theorem is verified.

Ex 2) Verify mean Value theorm for the function $f(x) = x^2 - 4x - 3$ in $(1,4)$.

→ The mean value Theorem is given by

If $F(x)$ is continuous on $[a,b]$ and differentiable in (a,b), then there exist at least one Value c in (a,b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

Given $f(x) = x^2 - 4x - 3$ .... ① & $a = 1$, $b = 4$

Clearly given function $f(x) = x^2$ which is being a polynomial, is continuous $[1,4]$ and differentiable in $(1,4)$.

$f(a) = f(1) = (1)^2 - 4(1) - 3 = 1 - 4 - 3 = -6$

$f(b) = f(4) = (4)^2 - 4(4) - 3 = 16 - 16 - 3 = -3$

Differentiate (1) with respect to x, we get

$f'(x) = 2x - 4 \Rightarrow f'(c) = 2c - 4$

Let $f'(c) = \frac{f(b)-f(a)}{b-a} \Rightarrow 2c - 4 = \frac{f(4)-f(1)}{4-1}$

$\Rightarrow 2c - 4 = \frac{-3-(-6)}{3} \Rightarrow 2c - 4 = \frac{-3+6}{3} \Rightarrow 2c - 4 = 1$

$\Rightarrow 2c = 5 \Rightarrow c = \frac{5}{2} = 2.5$

$\therefore c = 2.5 \in (1,4)$ Such that $f'(2.5) = 1 = \frac{f(4)-f(2)}{4-2}$

Hence mean value theorem is verified.

* Cauchy's Mean Value Theorem

Statement - If $f(x)$ and $g(x)$ are continuous in $[a,b]$, are differentiable in (a,b) and $g'(x) \neq 0$ for any value of x in (a,b), then there exist at least one value c in (a,b) such that $\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$

Ex-1) Verify Cauchy's Means Value Theorem for the functions $f(x) = x^3$ and $g(x) = x^2$ in $[0,2]$

Solution - The Cauchy's Mean Value Theorem is given by

If $f(x)$ & $g(x)$ are continuous in $[a,b]$, are differentiable in (a,b) and $g'(x) \neq 0$ for any ~~variable~~ value of x in (a,b), then there exist at least one value c in (a,b) such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$$

Given $f(x) = x^3$ & $g(x) = x^2$ which are being polynomials, are continuous in $[0,2]$ and are differentiable in $(0,2)$.

$f(a) = f(0) = (0)^3 = 0$ $\quad$ $g(a) = g(0) = (0)^2 = 0$

$f(b) = f(2) = (2)^3 = 8$ $\quad$ $g(b) = g(2) = (2)^2 = 4$

Differentiate $f(x)$ & $g(x)$ with respect to x, we get

$f'(x) = 3x^2$ $\quad$ $g'(x) = 2x$ $\quad$ Clearly $g'(x) \neq 0$ for any value of x in $(0,2)$.

$f'(c) = 3c^2$ $\quad$ $g'(c) = 2c$

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Let $\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)} \Rightarrow \frac{3c^2}{2c} = \frac{f(2)-f(0)}{g(2)-g(0)} \Rightarrow \frac{3c}{2} = \frac{8-0}{4-0}$

$\Rightarrow \frac{3c}{2} = 2 \Rightarrow c = \frac{4}{3}$

$\therefore c = 1.3334 \in (0,2)$ such that $\frac{f'(\frac{4}{3})}{g'(\frac{4}{3})} = 2 = \frac{f(2)-f(0)}{g(2)-g(0)}$

Hence Cauchy's mean value theorem is Verified.

Excellent

Seen 05/10/22

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5/12/23
Tuesday

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Assignment -3

Q1) State Rolle's theorem and verify the same for $f(x) = x^2 + 2x - 8$ in $(-4, 2)$

→ Statement : If $f(x)$ is continuous on $[a,b]$ and differentiable in (a,b), such that $f(a) = f(b)$, where a & b are some real numbers, then there exist at least one value c in (a,b) such that $f'(c) = 0$.

(i) $f(x) = x^2 + 2x - 8$ in $(-4, 2)$

Solution: The Rolle's theorem is given by

"If $f(x)$ is continuous on $[a,b]$ and differentiable in (a,b), such that $f(a) = f(b)$ where a & b are some real numbers, then there exist at least one value c in (a,b) such that $f'(c) = 0$."

Given $f(x) = x^2 + 2x - 8$ .....① & $a = -4$, $b = 2$

Clearly given function $f(x) = x^2 + 2x - 8$ which is being a polynomial, is continuous $[-4, 2]$ and differentiable in $(-4, 2)$.

$$f(a) = f(-4) = (-4)^2 + 2(-4) - 8 = 0$$

$$f(b) = f(2) = (2)^2 + 2(2) - 8 = 0$$

$$\therefore f(-4) = f(2) \text{ i.e., } f(a) = f(b)$$

Differentiate (i) with respect to x, we get

$$f'(x) = 2x + 2(1) + 0 = 2x + 2 \Rightarrow f'(x) = 2x + 2$$

$$\therefore f'(c) = 2c + 2$$

Let $f'(c) = 0 \Rightarrow 2c + 2 = 0 \Rightarrow 2c = -2 \Rightarrow c = -\frac{2}{2} \Rightarrow c = -1$

$\therefore c = -1 \in (-4, 2)$ such that $f'(c) = f'(-1) = 2(-1) + 2 = 0$

Hence Rolle's theorem is verified.

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Q2) State Lagrange's Mean value theorem. Verify Lagrange's Mean Value theorem for

(1) $f(x) = x^2$ in $(2,4)$

→ The mean value theorem is given by

If $f(x)$ is continuous on $[a,b]$ and differentiable in (a,b), then there exist at least one value c in (a,b)

Such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

Given $f(x) = x^2$ — ① & $a = 2$, $b = 4$

Clearly given function $f(x) = x^2$ which is being a polynomial, is continuous $[2,4]$ and differentiable in $(2,4)$.

$$f(a) = f(2) = (2)^2 = 4$$
$$f(b) = f(4) = (4)^2 = 16$$

Differentiate (1) with respect to x, we get

$$f'(x) = 2x \Rightarrow f'(c) = 2c$$

let $f'(c) = \dfrac{f(b)-f(a)}{b-a} \Rightarrow 2c = \dfrac{f(4)-f(2)}{4-2} \Rightarrow 2c = \dfrac{16-4}{2} \Rightarrow 2c = 6 \Rightarrow c = 3$

$\therefore c = 3 \in (2,4)$ Such that $f'(3) = 6 = \dfrac{f(4)-f(2)}{4-2}$

Hence Lagrange's mean value theorem is verified.

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(ii) $f(x) = x^2 - 4x - 3$ in $(1,4)$

→ The Mean value Theorem is given by
If $f(x)$ is continuous on $[a,b]$ and differentiable in (a,b), then there exist at least one value c in (a,b) Such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

Given $f(x) = x^2 - 4x - 3$ —— ① & $a=1$, $b=4$

Clearly given function $f(x) = x^2$ which is being a polynomial, is continuous $[1,4]$ and differentiable in $(1,4)$.

$$f(a) = f(1) = (1)^2 - 4(1) - 3 = 1 - 4 - 3 = -6$$

$$f(b) = f(4) = (4)^2 - 4(4) - 3 = 16 - 16 - 3 = -3$$

Differentiate (1) with respect to x, we get

$$f'(x) = 2x - 4 \Rightarrow f'(c) = 2c - 4$$

Let $f'(c) = \frac{f(b)-f(a)}{b-a} \Rightarrow 2c - 4 = \frac{f(4)-f(1)}{4-1}$

$\Rightarrow 2c - 4 = \frac{-3-(-6)}{3} \Rightarrow 2c - 4 = \frac{-3+6}{3} \Rightarrow 2c - 4 = 1 \Rightarrow$

$2c = 5 \Rightarrow c = \frac{5}{2} = 2.5$

$\therefore$ $c = 2.5 \in (1,4)$ Such that $f'(2.5) = 1 = \frac{f(4) - f(2)}{4-2}$

Hence mean Value theorem is verified.

(iii) $f(x) = x(x-1)$ in $(1,2)$

→ The Mean value Theorem is given by

If $f(x)$ is continuous on $[a,b]$ and differentiable in (a,b), then there exist at least one value c in (a,b) such that $f'(c) = \frac{f(b) - f(a)}{b-a}$

Given $f(x) = x^2 - x$ — ① & $a=1$, $b=2$

Clearly given function $f(x) = x^2$ which is being a polynomial, is continuous $[1,2]$ and differentiable in $(1,2)$

$f(a) = f(1) = (1)^2 - (1) = 1-1 = 0$

$f(b) = f(2) = (2)^2 - (2) = 4-2 = 2$

Differentiate (1) with respect to x, we get

$f'(x) = 2x - 1 \Rightarrow f'(c) = 2c-1$

Let $f'(c) = \frac{f(b) - f(a)}{b-a} \Rightarrow 2c-1 = \frac{f(2) - f(1)}{2-1}$

$\Rightarrow 2c - 1 = \frac{2-0}{1} \Rightarrow 2c-1 = 2 \Rightarrow 2c = 2+1 \Rightarrow 2c = 3 \Rightarrow c = \frac{3}{2}$
$= 1.5$

$\therefore a = 1.5 \in (1,2)$ such that $f'(1.5) = 2 = \frac{f(4) - f(2)}{4-2}$

Hence mean value theorem is verified.

Q3) State cauchy's Mean Value theorem, Verify cauchy's mean Value theorem for
(i) $f(x) = x^3$ and $g(x) = x^2$ in $(0,2)$
→

Statement : If $f(x)$ & $g(x)$ are continuous in $[a,b]$, are differentiable in (a,b) and $g'(x) \neq 0$ for any value of x in (a,b), then there exist at least one value c in (a,b) Such that $\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$

(i) $f(x) = x^3$ and $g(x) = x^2$ in $(0,2)$
→ The Cauchy's Mean Value Theorem is given by

If $f(x)$ & $g(x)$ are continuous in $[a,b]$ are differentiable in (a,b) and $g'(x) \neq 0$ for any value of x in (a,b), then there exist at least one value c in (a,b) such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$$

Given $f(x) = x^3$ & $g(x) = x^2$ & $a=0$, $b=2$

Clearly given functions $f(x) = x^3$ & $g(x) = x^2$ which are being polynomials, are continuous in $[0,2]$ and are differentiable in $(0,2)$.

$$f(a) = f(0) = (0)^3 = 0 \qquad g(a) = g(0) = (0)^2 = 0$$
$$f(b) = f(2) = (2)^3 = 8 \qquad g(b) = g(2) = (2)^2 = 4$$

Differentiate $f(x)$ & $g(x)$ with respect to x, we get.

$f'(x) = 3x^2$ $g'(x) = 2x$ Clearly $g'(x) \neq 0$ for any value of x in $(0,2)$.

$f'(c) = 3c$ $g'(c) = 2c$

Let $\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)} \Rightarrow \frac{3c^2}{2c} = \frac{f(2)-f(0)}{g(2)-g(0)} \Rightarrow \frac{3c}{2} = \frac{8-0}{4-0}$

$\Rightarrow \frac{3c}{2} = 2 \Rightarrow c = \frac{4}{3}$

$\therefore c = 1.3334 \in (0,2)$ such that $\frac{f'(\frac{4}{3})}{g'(\frac{4}{3})} = 2$

$= \frac{f(2)-f(0)}{g(2)-g(0)}$

Hence Cauchy's mean value theorem is verified.

(2) $f(x) = x^2$ and $g(x) = x^4$ in $(0,1)$

→ The Cauchy's Mean Value Theorem is given by

If $f(x)$ & $g(x)$ are continuous in $[a,b]$, are differentiable in (a,b) and $g'(x) \neq 0$ for any value of x in (a,b), then there exist at least one value c in (a,b) such that $\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$

Given $f(x) = x^2$ & $g(x) = x^4$ & $a=0$, $b=1$

Clearly given functions $f(x) = x^2$ & $g(x) = x^4$ which are being polynomials, are continuous in $[0,1]$ and are differentiable in $(0,1)$

$f(a) = f(0) = (0)^2 = 0$ $g(a) = g(0) = (0)^4 = 0$

$f(b) = f(1) = (1)^2 = 1$ $g(b) = g(1) = (1)^4 = 1$

Differentiate f(x) & g(x) with respect to x, we get

$f'(x) = 2x$ $g'(x) = 4x^3$ clearly $g'(x) \neq 0$ for any value of x in (0,1)

$f'(c) = 2c$ $g'(c) = 4c^3$

$$\text{Let } \frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)} \Rightarrow \frac{2c}{4c^3} = \frac{f(1)-f(0)}{g(1)-g(0)}$$

$$\Rightarrow \frac{1}{2c^2} = \frac{1-0}{1-0} \Rightarrow 2c^2 = 1 \Rightarrow c^2 = \frac{1}{2} \therefore c = \pm\frac{1}{\sqrt{2}} = \pm 0.7071$$

$\therefore c = \pm 0.7071 \in (0,1)$ such that $\frac{f'(\frac{1}{\sqrt{2}})}{g'(\frac{1}{\sqrt{2}})} = 1 = \frac{f(1)-f(0)}{g(1)-g(0)}$

Hence Cauchy's mean value theorem is verified.

(III) $f(x) = x^2$ and $g(x) = x^3$ in (1,2)

→ The Cauchy's Mean Value Theorem is given by
If f(x) & g(x) are continuous in [a,b], are differentiable in (a,b) and $g'(x) \neq 0$ for any value of x in (a,b), then there exist at least one value c in (a,b) such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$$

Given $f(x) = x^2$ & $g(x) = x^3$ & $a = 1$, $b = 2$

Clearly given functions $f(x) = x^2$ & $g(x) = x^3$ which are being

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polynomials, are continuous in $[1,2]$ and are differentiable in $(1,2)$.

$f(a) = f(1) = (1)^2 = 1$ $\quad g(a) = g(1) = (1)^3 = 1$
$f(b) = f(2) = (2)^2 = 4$ $\quad g(b) = g(2) = (2)^3 = 8$

Differentiate $f(x)$ & $g(x)$ with respect to x, we get

$f'(x) = 2x \quad g'(x) = 3x^2$ $\quad$ clearly $g'(x) \neq 0$ for any value
$f'(c) = 2c \quad g'(c) = 3c^2$ $\quad$ of x in $(1,2)$

Let $\dfrac{f'(c)}{g'(c)} = \dfrac{f(b) - f(a)}{g(b) - g(a)} \Rightarrow \dfrac{2c}{3c^2} = \dfrac{f(2) - f(1)}{g(2) - g(1)} \Rightarrow \dfrac{2}{3c} = \dfrac{4-1}{8-1}$

$\Rightarrow \dfrac{2}{3c} = \dfrac{3}{7} \Rightarrow 9c = 14 \therefore c = \dfrac{14}{9} = 1.5556$

$\therefore c = 1.5556 \in (1,2)$ Such that $\dfrac{f'\left(\frac{14}{9}\right)}{g'\left(\frac{14}{9}\right)} = \dfrac{3}{7} = \dfrac{f(2) - f(1)}{g(2) - g(1)}$

~~$\therefore c = 1.5556 \notin (1,2)$ Such that~~

Hence Cauchy's mean value theorem is verified.

Excellent

Checked 08/12/23

8/12/23
Friday

Unit-4 Integral Calculus

1)(i) Introduction :- The Symbol $\boxed{\int dx}$ is Know as integral operator. If f(x) is a function then its integration is given as $\int f(x)\,dx = F(x)+c$ where c is Known as constant of integration.

where; f(x) is Known as Integrand.

F(x) is Known as Integral.

Thus the process of finding the integral from integrand, is known as integration. Sometimes integration is termed as Antiderivative.

(ii) Some Standard Integral's

(a) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

(b) $\int \frac{1}{x} dx = \log x + c$

(c) $\int e^x dx = e^x + c$

(d) $\int \sin x \, dx = -\cos x + c$

(e) $\int \cos x \, dx = \sin x + c$

(f) $\int \sec^2 x \, dx = \tan x + c$

(g) $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

$\int(x \sin x)\,dx$

(h) $\int \sec x \tan x\, dx = \sec x + c$

(i) $\int \operatorname{cosec} x \cot x = -\operatorname{cosec} x + c$

Examples - find

(1) $\int (x^2+2x+1)\,dx$

$\longrightarrow \int (x^2+2x+1)\,dx = \int x^2 dx + \int 2x\,dx + \int 1\,dx$

$= \frac{x^{2+1}}{2+1} + 2\int x^1 dx + \int x^0 dx$

$= \frac{x^3}{3} + \frac{2x^{1+1}}{1+1} + \frac{x^{0+1}}{0+1} + c$

$= \frac{x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} + \frac{x}{1} + c$

$$\boxed{\frac{x^3}{3} + x^2 + x + c}$$

(2) $\int (\log x \cancel{+} x^5)\,dx$

$\longrightarrow \int (\log x + x^5)\,dx = \int \log x\, dx + \int x^5 dx$

$= \frac{1}{x} \cancel{+}$

(2) $\int \left(\frac{1}{x} + x^5\right) dx$

$\longrightarrow \int \left(\frac{1}{x} + x^5\right) dx = \int \frac{1}{x} dx + \int x^5 dx$

$= \log x + 5 \int x^5 dx$

$= \log x + 5 x^{4+1}$

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(2) $\int\left(\frac{1}{x} + x^5\right) dx$

$\longrightarrow \int\left(\frac{1}{x} + x^5\right) dx = \int \frac{1}{x} dx + \int x^5 dx$

$= \log x + \frac{x^{5+1}}{5+1} + c$

$= \log x + \frac{x^6}{6} + 6$

8/12/18
Friday

1) $\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)+c$

2) $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)+c$

3) $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$

4) $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right)+c$

5) $\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left(x+\sqrt{x^2-a^2}\right)+c$

6) $\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \log\left(x+(\sqrt{x^2+a^2}\right)+c$

7) $\int e^{ax}\sin bx\,dx = \left(\frac{e^{ax}}{a^2+b^2}\right)(a\sin bx - b\cos bx)+c$

8) $\int e^{ax}\cos bx\,dx = \left(\frac{e^{ax}}{a^2+b^2}\right)(a\cos bx + b\sin bx)+c$

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* Solved Problems

Integrate the following with respect to x

(1) x^6

$\longrightarrow \int x^6 dx = \frac{x^{6+1}}{6+1} + c = \frac{x^7}{7} + c$

(2) $\int x^{-2} dx = \frac{x^{-2+1}}{-2+1} + c = \frac{-1}{x} + c$

(3) $\int \frac{1}{x^{10}} dx = \int x^{-10} dx = \left(\frac{-1}{9}\right)\frac{1}{x^9} + c$

(4) $\sqrt{x}$

$\int \sqrt{x}\, dx = \int x^{1/2} dx = \frac{x^{3/2}}{(3/2)} + c = \frac{2}{3} x^{3/2} + c$

(5) $\frac{1}{\sqrt{x}}$

$\longrightarrow \int \frac{1}{\sqrt{x}} dx = \int \frac{1}{x^{1/2}} dx = \int x^{-1/2} dx = \frac{x^{1/2+1}}{-\frac{1}{2}+1} + c$

$= \frac{x^{1/2}}{(1/2)} + c = 2\sqrt{x} + c$

(6) $\frac{1}{\sqrt[3]{x^5}}$

$\longrightarrow \int \frac{1}{\sqrt[3]{x^5}} dx = \int \frac{1}{x^{5/3}} dx$

$= \int x^{-5/3} dx = \frac{x^{-5/3+1}}{\left(-\frac{5}{3}+1\right)} + c$

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$$= \frac{x^{-2/3}}{(-2/3)} + c$$

$$= \left(\frac{-3}{2}\right) \frac{1}{x^{2/3}} + c.$$

⟨7⟩ $\frac{1}{\sin^2 x}$

$\rightarrow \int \frac{1}{\sin^2 x}\, dx = \int \operatorname{cosec}^2 x\, dx = -\cot x + c$

(8) $\frac{\sin x}{\cos^2 x}$

$\rightarrow \int \frac{\sin x}{\cos^2 x}\, dx = \int \left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\cos x}\right) dx$

$= \int \tan x \sec x\, dx$

$= \sec x + c$

(9) $\frac{\cot x}{\sin x}$

$\rightarrow \int \frac{\cot x}{\sin x}\, dx = \int \left(\frac{1}{\sin x}\right) \cot x\, dx$

$= \int \operatorname{cosec} x \cot x\, dx$

$= -\operatorname{cosec} x + c$

(10) $\frac{1}{x^2+16}\, dx$

$\rightarrow \int \frac{1}{x^2+16}\, dx = \int \frac{1}{x^2+4^2}\, dx$

$= \frac{1}{4} \tan^{-1}\left(\frac{x}{4}\right) + c$

$\frac{1}{\sqrt{25-x^2}}$

(11) $\int \frac{1}{\sqrt{25-x^2}} dx = \int \frac{1}{\sqrt{5^2-x^2}} dx$

$= \sin^{-1}\left(\frac{x}{5}\right)+c$

(12) $\frac{1}{\sqrt{x^2-9}} dx$

$\longrightarrow \int \frac{1}{\sqrt{x^2-9}} dx = \int \frac{1}{\sqrt{x^2-3^2}} dx$

$= \log\left[x+\sqrt{x^2-9}\right]+c$

(13) $e^{2x}\sin 3x$ $\left[\text{Here } a=2 \text{ and } b=3\right]$

$\longrightarrow \int e^{2x}\sin 3x\, dx$

$= \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)+c$

$= \frac{e^{2x}}{13}(2\sin 3x - 3\cos 3x)+c$

(14) $e^{x}\cos 2x$ $\left[\text{Here } a=1 \text{ and } b=2\right]$

$\longrightarrow \int e^{x}\cos 2x\, dx$

$= \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)+c$

$= \frac{e^{x}}{5}(\cos 2x + 2\sin 2x)+c$

10/12/23
Tuesday

Jānhavi shukla
B.C.A - 9944

Assignment - 4

INTEGRAL CALCULUS

Q1) Integrate the following with respect to x^{16}

$\rightarrow \int x^{16} dx$

we know that ;

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{16} dx = \frac{x^{16+1}}{16+1} + c$$

$$\therefore = \boxed{\frac{16 x^{17}}{17} + c}$$

Q2] $\frac{1}{x^3}$

$\rightarrow \int \frac{1}{x^3} dx$

we know that : $\int \frac{1}{x^n} dx = \left(\frac{-1}{n-1}\right)\left(\frac{1}{x^{n-1}}\right) + c$

$$\therefore \int \frac{1}{x^3} dx = \boxed{\left(\frac{-1}{4}\right)\left(\frac{1}{x^3}\right) + c}$$

c) $3\sqrt{x^4}$

$\rightarrow \int \sqrt[3]{x^4} dx$

$= (x^4)^{3/2} dx$

$= x^{12/2}$

$= \boxed{x^6 + c}$

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(d) $\frac{1}{e^x}$

$\longrightarrow$ $\int \frac{1}{e^{-x}}\, dx$

$\therefore \boxed{e^x + c}$

(e) $\frac{1}{\cos^2 x}$

$\longrightarrow$ $\int \frac{1}{\cos^2 x}\, dx$

$\int \frac{1}{\cos^2 x}\, dx = \cos^{-2} x + c$

$= \boxed{\tan x + c}$

$\operatorname{cosec} = \frac{1}{\sin x}$

$\sec x = \frac{1}{\cos x}$, i.e. $\cos x$

$\int \sec^2 x\, dx = \tan x + t$

$\frac{d}{dx}(\tan x) = \sec^2 x$

(f) $\frac{\tan x}{\cos x}$

$\longrightarrow$ $\int \frac{\tan x}{\cos x}\, dx$

$\int \frac{\tan x}{\cos x}\, dx = \boxed{\sec x + c}$

(g) $\frac{\cos x}{\sin^2 x}$

$\longrightarrow$ $\int \frac{\cos x}{\sin^2 x}\, dx$

$\int \frac{\cos x}{\sin^2 x}\, dx = \boxed{-\operatorname{cosec} x + c}$

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(h) $\frac{1}{25+x^2}$

$\rightarrow \int \frac{1}{25+x^2}$

$\int \frac{1}{25+x^2}$ $\boxed{dx = \frac{1}{5}\tan^{-1}(x/5) + c}$

(i) $\frac{1}{\sqrt{36-x^2}}$

$\rightarrow \int \frac{1}{\sqrt{36-x^2}}\,dx$

$\int \frac{1}{\sqrt{36-x^2}}$ $\boxed{dx = \sin^{-1}(x/6) + c}$

(j) $\frac{1}{\sqrt{9+x^2}}$

$\rightarrow \int \frac{1}{\sqrt{9+x^2}}\,dx$

$\int \frac{1}{\sqrt{9+x^2}}$ $\boxed{dx = \log(x+\sqrt{x^2+9})}$

(k) $e^{3x}\sin 2x$

$\rightarrow \int e^{3x}\sin 2x\,dx$

$\int e^{3x}\sin 2x$ $\boxed{dx = \frac{e^{3x}}{13}(3\sin 2x - 2\cos 2x)}$

Excellent

Checked 12/12/23

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Tuesday

QUESTION BANK

Q1> $\frac{d}{dx}(c) = 0$

Q2> $\frac{d}{dx}(e^x) = e^{ax} \cdot a$

Q3> $\frac{d}{dx}(\log x) = \cancel{n x^{n-1}}$

Q4> $\frac{d}{dx}(x^n) = n x^{n-1}$

Q5> $\frac{d}{dx}(\sin x) = \cos x$

Q6> $\frac{d}{dx}(\cos x) = -\sin x$

Q7> $\frac{d}{dx}(\tan x) = \sec^2 x$

Q8> Define integration of a function $f(x)$

→ The Symbol $\int dx$ is known as integral operator. If $f(x)$ is a function then its integration is given as $\int f(x)\, dx = f(x) + c$, where c is known as constant of integration.

where; $f(x)$ is known as Integrand.
$f(x)$ is known as Integral.

Thus the process of finding the integral from integrand, is known as integration.

$\frac{19}{1}/81$

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Q9) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Q10) $\int \frac{1}{x} dx = \log x + c$

Q11) $\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log\left(\frac{x-a}{x+a}\right) + c$

Q12) $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log\left(\frac{a+x}{a-x}\right) + c$

Q13) $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$

Q14) $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left(x + \sqrt{x^2 - a^2}\right) + c$

Q15) $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

Q16) $\int \frac{1}{\sqrt{x^2 + a^2}} dx = \log\left(x + \sqrt{x^2 + a^2}\right) + c$

Q17) The co-ratio of $\sec\theta$ is $\operatorname{cosec}\theta$

Q18) $45° = \frac{\pi}{4}$ radians

Q19) The reciprocal ratio of $\tan\theta$ is $\cot\theta$

Q20) The value of $\sin(120)$ is $\frac{\sqrt{3}}{2}$

Q21) $\sin(A+B) = \sin A \cos B + \cos A \sin B$.

Q22) Find transpose of the matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ 12 & 41 & -6 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 12 \\ -2 & 41 \\ 1 & -6 \end{bmatrix}$$

Q23) If $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ then its determinant is -1

Q24) Write an example of a skew-Symmetric matrix.

$\longrightarrow = \begin{bmatrix} 0 & 11 & -12 \\ -11 & 0 & 4 \\ 2 & -4 & 0 \end{bmatrix}$

Q25) Define the term "orthogonal matrix".

$\longrightarrow$ If $A'A = AA' = I$, then matrix A is called an orthogonal matrix.

Q26) If $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ then order of A.B is 2×2

Q27) 3 radians $= \frac{540}{\pi}$ degrees.

Q28) $15^\circ = \frac{\pi}{12}^c$

Q29) The reciprocal ratio of $\tan\theta$ is $\cot\theta$

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Q30) The value of $\sin(270-\theta)$ is $-\cos\theta$

Q31) $\cos(A+B) = \cos A \cos B - \sin A \sin B$.

Q32) The order of the matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 6 \end{bmatrix}$ is 2×3

Q33) If $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ then its transpose is $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

Q34) Write an example of a Symmetric matrix.

→ $A' = A$

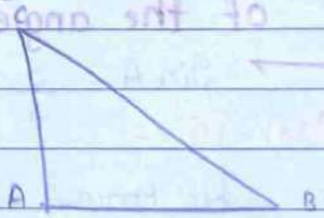
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 6 \\ 3 & 6 & 9 \end{bmatrix} = A' = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

Q35) Define the term "Singular matrix".

→ The determinant of a matrix is a real number and is denoted as $|A| = 0$. if $|A| = 0$, then A is said to be a Singular matrix.

Q36) If $A = \begin{bmatrix} 3 & 4 \\ 7 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then $A+B$ is $\begin{bmatrix} 4 & 6 \\ 10 & 12 \end{bmatrix}$

Q37) Find the remaining trigonometric ratios of the angle A, if $\tan A = \frac{4}{3}$



→ $\tan A = \frac{4}{3}$

$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{o}{a} = \frac{4}{3}$

Here, $o = 4$ and $a = 3$, $h = ?$

By pythagoras theorem

$$h^2 = o^2 + a^2$$

$$\Rightarrow h = \sqrt{o^2 + a^2}$$

$$\Rightarrow h = \sqrt{(4)^2 + (3)^2}$$

$$\Rightarrow h = \sqrt{16+9} = \sqrt{25} = 5$$

$\therefore h = 5$

Now,

$$\sin A = \frac{o}{h} = \frac{4}{5}$$

$$\cos A = \frac{a}{h} = \frac{3}{5}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{5}{4}$$

$$\sec A = \frac{1}{\cos A} = \frac{5}{3}$$

$$\cot A = \frac{1}{\tan A} = \frac{3}{4}$$

Hence, other trigonometric ratios are.

$\sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$, $\operatorname{cosec} A = \frac{5}{4}$, $\sec A = \frac{5}{3}$, $\cot = \frac{3}{4}$

Q38) If $\sin A = \frac{3}{5}$ find the remaining trigonometric ratios of the angle A.

→ $\sin A = \frac{3}{5}$

We know, $\sin A = \frac{\text{apposite}}{\text{hypo}} = \frac{o}{h} = \frac{3}{5}$

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Here, $o = 3$ and $h = 5$, $a = ?$

By pythagoras theorem

$$h^2 = o^2 + a^2$$
$$\Rightarrow h^2 - o^2 = a^2$$
$$\Rightarrow a = \sqrt{h^2 - o^2}$$
$$\Rightarrow a = \sqrt{(5)^2 - (3)^2}$$
$$\Rightarrow a = \sqrt{125 - 9}$$
$$\Rightarrow a = \sqrt{16}$$
$$\therefore a = 4$$

Now,

$$\cos A = \frac{a}{h} = \frac{4}{5}$$

$$\tan A = \frac{o}{a} = \frac{3}{4}$$

$$\cot A = \frac{1}{\tan A} = \frac{4}{3}$$

$$\operatorname{Cosec} A = \frac{1}{\sin A} = \frac{5}{3}$$

$$\sec A = \frac{1}{\cos A} = \frac{5}{4}$$

Ans = $\cos A = \frac{4}{5}$, $\tan A = \frac{3}{4}$, $\cot A = \frac{4}{3}$, $\operatorname{cosec} A = \frac{5}{3}$ and $\sec A = \frac{5}{4}$

Q39) Find the determinant of the matrix $B = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$\longrightarrow = 2(6-2) - 2(2-1) + 1(2-3)$

$= 2(4) - 2(1) + 1(-1)$

$= 8 - 2 - 1$

$= \underline{5}$

Q40) What is an orthogonal matrix and show that the matrix $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ is an orthogonal matrix.

→ If $AA' = A'A = I$, then matrix A is called as orthogonal matrix.

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AA' = I$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore AA' = I$$

Hence, matrix $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ is an orthogonal matrix.

Q41) Find the determinant of the matrix $P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 3 & 11 \\ 1 & 0 & -2 \end{bmatrix}$

→ $P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 3 & 11 \\ 1 & 0 & -2 \end{bmatrix}$

$= 1(-6-0) - 0 + 1(0-3)$

$= 1(-6) + 1(-3)$

$= -6 - 3$

$= -9$

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Q42) If $\tan C = \frac{3}{5}$, find the remaining trigonometric ratios of the angle C.

→ $\tan C = \frac{3}{5}$

We know,

$\tan C = \frac{\text{oppo}}{\text{adja}} = \frac{o}{a} = \frac{3}{5}$

Here $o = 3$ and $a = 5$, $h = ?$

By pythagoras theorem

$\Rightarrow h^2 = o^2 + a^2$

$\Rightarrow h = \sqrt{o^2 + a^2}$

$\Rightarrow h = \sqrt{(3)^2 + (5)^2}$

$\Rightarrow h = \sqrt{9 + 25}$

$\Rightarrow h = \sqrt{34}$

$\boxed{h = \sqrt{34}}$

Now,

$\sin C = \frac{3}{\sqrt{34}}$

$\cos C = \frac{5}{\sqrt{34}}$

$\cot C = \frac{1}{\tan C} = \frac{5}{3}$

$\operatorname{cosec} C = \frac{1}{\sin C} = \frac{\sqrt{34}}{3}$

$\sec C = \frac{1}{\cos C} = \frac{\sqrt{34}}{5}$

Q43) Find the Value of A^2, where $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & 4 & 2 \end{bmatrix}$

→ $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & 4 & 2 \end{bmatrix}$

$$A^2 = AA \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 1+0+3 & 2+2+4 & 1-2+2 \\ 0+0-3 & 0+1-4 & 0-1-2 \\ 3+0+6 & 6+4+8 & 3-4+4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4 & 8 & 1 \\ -3 & -3 & -3 \\ 9 & 18 & 3 \end{bmatrix}$$

Q44) Find $(A+B)$ and $(A.B)$, where $A = \begin{bmatrix} 1 & 1 & 3 \\ -1 & 0 & 2 \\ -2 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 4 & 1 \\ 2 & 0 & 1 \end{bmatrix}$.

→

$$(A+B) = \begin{bmatrix} 1 & 1 & 3 \\ -1 & 0 & 2 \\ -2 & 1 & 5 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 3 \\ 1 & 4 & 1 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 4 & 3 \\ 0 & 1 & 6 \end{bmatrix}$$

$$(A.B) = \begin{bmatrix} 1 & 1 & 3 \\ -1 & 0 & 2 \\ -2 & 1 & 5 \end{bmatrix} . \begin{bmatrix} 0 & -1 & 3 \\ 1 & 4 & 1 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0+1+6 & -1+4+0 & 3+1+3 \\ 0+0+4 & 1+0+0 & -3+0+2 \\ 0+1+10 & 2+4+0 & -6+1+5 \end{bmatrix}$$

$$(A.B) = \begin{bmatrix} 7 & 3 & 7 \\ 4 & 1 & -1 \\ 11 & 6 & 0 \end{bmatrix}$$

Q45) Solve the System of equations given below by using Gauss Jordan method

(a) $x + y + z = 9$

(b) $x - 2y + 3z = 8$

(c) $2x + y - z = 3$

→ $AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ 3 \end{bmatrix}$$

Let $[A:B]$ $\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 1 & -2 & 3 & 8 \\ 2 & 1 & -1 & 3 \end{array}\right]$ is a agumented matrix

$R_2 \to R_2 - R_1$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 2 & 1 & -1 & 3 \end{array}\right]$$

$R_3 \to R_3 - 2R_1$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & -1 & -3 & -15 \end{array}\right]$$

$R_3 \to R_2 - 3R_3$

$R_3 \to R_2 - 3R_3$

$-3 - 3(-1)$

$-3 + 3$

0

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 11 & 44 \end{array}\right]$$

$\frac{11}{11}$

$\frac{0}{11}$

0

$R_3 \to \frac{R_3}{11}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$R_1 \rightarrow R_1 - R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$R_1 \rightarrow 3R_1 + R_2$

$$\left[\begin{array}{ccc|c} 3 & 0 & 0 & 6 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$\therefore \boxed{x=6}, \boxed{y=-9}$ and $\boxed{z=4}$

Q 46) Find the rank of A, by reducing into Echelon from, where $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

$\longrightarrow$ $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

$R_2 \rightarrow R_2 - R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -2 \\ 2 & 6 & 5 \end{bmatrix}$$

$R_2 \longrightarrow \frac{R_2}{2}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1/2 \\ 2 & 6 & 5 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1/2 \\ 0 & 2 & -1 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence, a rank of a matrix A is $\underline{\underline{2}}$

Q47) Reduce into the Echelon form and find the rank of a matrix $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

$\longrightarrow A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

$R_1 \longleftrightarrow R_2$

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - 3R_1$

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$$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - R_2$

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$R_4 \longrightarrow R_4 - R_2$

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$R_4 \longrightarrow R_4 - R_1$

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(A) = 2$

Q48) Solve the system of equations given below by using Gauss Elimination.

(a) $x+y+z=9$

(b) $x-2y+3z=8$

(c) $2x+y-z=3$

→ $AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ 3 \end{bmatrix}$$

Let $[A.B]$ $\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 1 & -2 & 3 & 8 \\ 2 & 1 & -1 & 3 \end{array}\right]$ is a agumented matrix

$R_2 \rightarrow R_2 - R_1$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 2 & 1 & -1 & 3 \end{array}\right]$$

$R_3 \rightarrow R_3 - 2R_1$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & -1 & -3 & -15 \end{array}\right]$$

$R_3 \rightarrow R_2 - 3R_3$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 11 & 44 \end{array}\right]$$

reverse equation :- $x+y+z = 9$ — ①

$-3y + 2z = -1$ — (ii)

$11z = 44$ — (iii)

$\boxed{z=4}$

put $z=4$ in equation (ii)

$-3y + 2(4) = -1$

$-3y + 8 = -1$

$-3y = -1 - 8$

$\cancel{-}3y = \cancel{-}9$

$\boxed{y=3}$

put $y=3$ and $z=4$ in eqn (i)

$x+3+4 = 9$

$x+7 = 9$

$\therefore \boxed{x=2}$

Hence, the value of x, y and z are $2, 3$ and 4 respectively.

50) Differentiate with respect x

(a) $f(x) = x^2+2$

$\rightarrow \frac{dy}{dx} f(x) = \frac{d}{dx}(x^2+2)$ $= \frac{d}{dx}x^2 + \frac{d}{dx}2$

$f'(x) = 2x$ $= 2x + 0$

$= 2x$

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$\sqrt[3]{x} = \underline{(x)}^{3/2}$

$\frac{1}{2\sqrt{x}}$

(b) $f(x) = 3\sqrt{x} - \frac{2}{x}$

$\longrightarrow \frac{d}{dx} f(x) = \frac{dy}{dx}\left(3\sqrt{x} - \frac{2}{x}\right)$

$f'(x) = \cancel{3}\frac{d}{dx} 3\sqrt{x} - \frac{d}{dx}\frac{2}{x}$

$= 3\frac{d}{dx}(x)^{1/2} - 2\frac{d}{dx}\frac{1}{x}$

$= 3\left(\frac{1}{2}x^{1/2-1}\right) - 2\frac{d}{dx}x^{-1}$

$= \frac{3}{2}x^{-1/2} - 2(-1)x^{-1-1}$

$= \frac{3}{2}\frac{1}{x^{1/2}} + 2x^{-2}$

$= \frac{3}{2}\frac{1}{\sqrt{x}} + 2\frac{1}{x^2}$

$= \frac{3}{2\sqrt{x}} + \frac{2}{x^2}$

(c) $f(x) = 2e^x - 4\log x$

$\longrightarrow f(x) = 2\frac{d}{dx}e^x - 4\frac{d}{dx}\log x$

$= 2e^x - 4\frac{1}{x}$

$\therefore 2e^x - \frac{4}{x}$

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(d) $f(x) = 2\sin x - 3\cos x$

$\longrightarrow f(x) = 2\frac{d}{dx}\sin x - 3\frac{d}{dx}\cos x$

$= 2\cos x - 3(-\sin x)$

$= 2\cos x + 3\sin x$

(e) $f(x) = 3\tan x - 4x$

$\longrightarrow f(x) = \frac{d}{dx}3\tan x - \frac{d}{dx}4x$

$= 3\frac{d}{dx}\tan x - 4\frac{d}{dx}x$

$= 3\sec^2 x - 4$

(f) $f(x) = 5x^2 + 2\cot x - 3\log x$

$\longrightarrow f(x) = \frac{d}{dx}5x^2 + \frac{d}{dx}2\cot x - \frac{d}{dx}3\log x$

$= 5\frac{d}{dx}x^2 + 2\frac{d}{dx}\cot x - 3\frac{d}{dx}\log x$

$= 5 \cdot 2x + 2(-\csc^2 x) - 3\frac{1}{x}$

$= 10x - 2\csc^2 x - \frac{3}{x}$

(g) $f(x) = 3\operatorname{cosec}x + \sec x$

→ $f(x) = \frac{d}{dx} 3\operatorname{cosec}x + \frac{d}{dx}\sec x$

$= 3\frac{d}{dx}\operatorname{cosec}x + \sec x \cdot \tan x$

$= 3(-\operatorname{cosec}x \cdot \cot x) + \sec x \tan x$

$= -3\operatorname{cosec}x \cdot \cot x + \sec x \tan x$

(h) $f(x) = 2x^2 \sin x$ (multiplication rule)

→ $f(x) = \frac{d}{dx} 2x^2 \sin x$

$= 2\frac{d}{dx} x^2 \sin x$

$= 4x\cos x$

Q51) State Rolle's theorem and verify the same for $f(x) = x^2+2x-8$ in $(-4,2)$

→ Statement - If $f(x)$ is continuous on $[a,b]$ and differentiable in (a,b), such that $f(a) = f(b)$, where a & b are some real numbers, then there exist at least one value c in (a,b) such that $f'(c) = 0$

(i) $F(x) = x^2+2x-8$ in $(-4,2)$

→ The Rolle's theorem is given by

"If $f(x)$ is continuous on $[a,b]$ and differentiable in (a,b), such that $f(a) = f(b)$ where a & b are some real numbers, then there exist at least one value c in (a,b) such that $f'(c) = 0$".

Given $f(x) = x^2 + 2x - 8$ —① & $a = -4, b = 2$

Clearly given Function $f(x) = x^2 + 2x - 8$ which is being a polynomial, is Continuous $[-4, 2]$ and differentiable in $(-4, 2)$.

$f(a) = f(-4) = (-4)^2 + 2(-4) - 8 = 0$

$f(b) = f(2) = (2)^2 + 2(2) - 8 = 0$

$\therefore f(-4) = f(2)$ i.e., $f(a) = f(b)$

Differentiable (1) with respect to x, we get

$f'(x) = 2x + 2(1) + 0 = 2x + 2 \Rightarrow f'(x) = 2x + 2$

$\therefore f'(c) = 2c + 2$

Let $f'(c) = 0 \Rightarrow 2c + 2 = 0 \Rightarrow 2c = -2 \Rightarrow c = -\frac{2}{2} \Rightarrow c = -1$

$\therefore c = -1 \in (-4, 2)$ such that $f'(c) = f'(-1)$

$= 2(-1) + 2 = 0$

Hence Rolle's theorem is verified.

Q52) State lagrange's Mean Value theorem. verify Lagrange's Mean Value theorem for

(1) $f(x) = x^2$ in $(2, 4)$

⟶ Statement : If $f(x)$ is continuous on $[a, b]$ and differentiable in (a, b), then there exist at least one value c in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

for $= x^2$ in $(2, 4)$

⟶ The mean value theorem is given by

If $f(x)$ is continuous on $[a, b]$ and differentiable in (a, b), then there exist at least one value c in (a, b)

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Such that $f'(c) = \frac{f(b) - f(a)}{b-a}$

Given $f(x) = x^2$ —① & $a=2$, $b=4$

Clearly given function $f(x) = x^2$ which is being a polynomial, is continuous $[2,4]$ and differentiable in $(2,4)$.

$$f(a) = f(2) = (2)^2 = 4$$
$$f(b) = f(4) = (4)^2 = 16$$

Differentiate (1) with respect to x, we get

$$f'(x) = 2x \Rightarrow f'(c) = 2c$$

let $f'(c) = \frac{f(b) - f(a)}{b-a} \Rightarrow 2c = \frac{f(4) - f(2)}{4-2} \Rightarrow 2c = \frac{16-4}{2}$

$\Rightarrow 2c = 6 \Rightarrow c = 3$

$\therefore c = 3 \in (2,4)$ Such that $f'(3) = 6 = \frac{f(4) - f(2)}{4-2}$

Hence Largrange's mean value theorem is verified

(2) $f(x) = x^2 - 4x - 3$ in $(1,4)$

⟶ The mean value theorem is given by

If $f(x)$ is continuous on $[a,b]$ and differentiable in (a,b), then there exist at least one value c in (a,b)

Such that $f'(c) = \frac{f(b) - f(a)}{b-a}$

Given $f(x) = x^2 - 4x - 3$ —① & $a=1$, $b=4$

Clearly given function $f(x) = x^2$ which is being a polynomial, is continuous $[1,4]$ and differentiable in $(1,4)$.

$f(a) = f(1) = (1)^2 - 4(1) - 3 = 1 - 4 - 3 = -6$

$f(b) = f(4) = (4)^2 - 4(4) - 3 = 16 - 16 - 3 = -3$

Differentiate (i) with respect to x, we get
$f'(x) = 2x - 4 \Rightarrow f'(c) = 2c - 4$

let $f'(c) = \frac{f(b) - f(a)}{b-a} \Rightarrow 2c - 4 = \frac{f(4) - f(1)}{4-1}$

$\Rightarrow 2c - 4 = \frac{-3-(-6)}{3} \Rightarrow 2c-4 = \frac{-3+6}{3} \Rightarrow 2c - 4 = 1 \Rightarrow$

$2c = 5 \Rightarrow c = \frac{5}{2} = 2.5$

$\therefore c = 2.5 \in (1,4)$ such that $f'(2.5) = 1 = \frac{f(4) - f(2)}{4-2}$

Hence mean value theorem is verified.

(iii) $f(x) = x(x-1)$ in $(1,2)$

$\longrightarrow$ The mean Value Theorem is given by

If $f(x)$ is continuous on $[a,b]$ and differentiable in (a,b), then there exist at least one value c in (a,b) such that $f'(c) = \frac{f(b) - f(a)}{b-a}$

Given $f(x) = x^2 - x$ — ① & $a=1$, $b=2$

Clearly given function $f(x) = x^2$ which is being a polynomial, is continuous $[1,2]$ and differentiable in $(1,2)$

$f(a) = f(1) = (1)^2 - (1) = 1-1 = 0$
$f(b) = f(2) = (2)^2 - (2) = 4-2 = 2$

differentiate (1) with respect to x, we get
$f'(x) = 2x-1 \Rightarrow f'(c) = 2c-1$

Let $f'(c) = \frac{f(b)-f(a)}{b-a} \Rightarrow 2c-1 = \frac{f(2)-f(1)}{2-1}$

$\Rightarrow 2c-1 = \frac{2-0}{1} \Rightarrow 2c-1 = 2 \Rightarrow 2c = 2+1 \Rightarrow 2c = 3 \Rightarrow c = \frac{3}{2}$

$= 1.5$

$\therefore a = 1.5 \in (1,2)$ Such that $f'(1.5) = 2 = \frac{f(4)-f(2)}{4-2}$

Hence mean value theorem is verified.

Q53) State cauchy's Mean value theorem. verify cauchy's Mean value theorem for.

$\rightarrow$ Statment :- If $f(x)$ & $g(x)$ are continuous in $[a,b]$, are differentiable in (a,b) and $g'(x) \neq 0$ for any value of x in (a,b), then there exist at least one value c in (a,b)

Such that $\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$

(1) $f(x) = x^3$ and $g(x) = x^2$ in $(0,2)$

$\rightarrow$ The cauchy's Mean value theorem is given by
If $f(x)$ & $g(x)$ are continuous in $[a,b]$ are differentiable in (a,b) and $g'(x) \neq 0$ for any vale of x in (a,b) then there exist at least one value c in (a,b) such that

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$$\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$$

Given $f(x) = x^3$ & $g(x) = x^2$ & $a=0$, $b=2$

Clearly given functions $f(x) = x^3$ & $g(x) = x^2$ which are being polynomials, are continuous in $[0,2]$ and are differentiable in $(0,2)$.

$f(a) = f(0) = (0)^3 = 0$ $g(a) = g(0) = (0)^2 = 0$
$f(b) = f(2) = (2)^3 = 8$ $g(b) = g(2) = (2)^2 = 4$

Differentiate $f(x)$ & $g(x)$ with respect to x, we get

$f(x) = 3x^2$ $g'(x) = 2x$ clearly $g'(x) \neq 0$ for any value of x in $(0,2)$.

$f'(c) = 3c$ $g'(c) = 2c$

let $\frac{f(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)} \Rightarrow \frac{3c^2}{2c} = \frac{f(2)-f(0)}{g(2)-g(0)} \Rightarrow \frac{3c}{2} = \frac{8-0}{4-0}$

$\Rightarrow \frac{3c}{2} = 2 \Rightarrow c = \frac{4}{3}$

$\therefore c = 1.3334 \in (0,2)$ Such that $\frac{f'\left(\frac{4}{3}\right)}{g'\left(\frac{4}{3}\right)} = 2$

$= \frac{f(2)-f(0)}{g(2)-g(0)}$

Hence Cauchy's mean value theorem is verified.

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(2) $f(x) = x^2$ and $g(x) = x^4$ in $(0,1)$

→ The Cauchy's Mean Value Theorm is given by

If $f(x)$ & $g(x)$ are Continuous in $[a,b]$, are differentiable in (a,b) and $g'(x) \neq 0$ for any Value of x in (a,b) then there exist at least one value c in (a,b) Such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$$

Given $f(x) = x^2$ & $g(x) = x^4$ & $a=0$, $b=1$

Clearly given functions $f(x) = x^2$ & $g(x) = x^4$ which are being polynomials, are continuous in $[0,1]$ and are differentiable in $(0,1)$.

$f(a) = f(0) = (0)^2 = 0$ $\quad g(a) = g(0) = (0)^4 = 0$

$f(b) = f(1) = (1)^2 = 1$ $\quad g(b) = g(1) = (1)^4 = 1$

Differentiate $f(x)$ & $g'(x)$ with respect to x, we get

$f'(x) = 2x$ $\quad g'(x) = 4x^3$ $\quad$ Clearly $g'(x) \neq 0$ for any value of x in $(0,1)$

$f'(c) = 2c$ $\quad g'(c) = 4c^3$

let $\dfrac{f'(c)}{g'(c)} = \dfrac{f(b)-f(a)}{g(b)\ g(a)} \Rightarrow \dfrac{2c}{4c^3} = \dfrac{f(1)-f(0)}{g(1)-g(0)}$

$\Rightarrow \dfrac{1}{2c^2} = \dfrac{1-0}{1-0} \Rightarrow 2c^2 = 1 \Rightarrow c^2 = \dfrac{1}{2}$ $\therefore c = \pm \dfrac{1}{\sqrt{2}} = \pm 0.7071$

$\therefore c = + 0.7071 \in (0,1)$ Such that $\dfrac{f'(\frac{1}{\sqrt{2}})}{g'(\frac{1}{\sqrt{2}})} = 1 = \dfrac{f(1)-f(0)}{g(1)-g(0)}$

Hence Cauchy's mean value theorem is verified.

(3) $f(x) = x^2$ and $g(x) = x^3$ in $(1,2)$

→ The Cauchy's Mean value Theorem is given by
If $f(x)$ & $g(x)$ are continuous in $[a,b]$, are differentiable in (a,b) and $g'(x) \neq 0$ for any value of x in (a,b), then there exist at least one value c in (a,b) such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

Given $f(x) = x^2$ & $g(x) = x^3$ & $a=1$, $b=2$

Clearly given functions $f(x) = x^2$ & $g(x) = x^3$ which are being polynomials, are continuous in $[1,2]$ and are differentiable in $(1,2)$.

$f(a) = f(1) = (1)^2 = 1$ $\quad$ $g(a) = g(1) = (1)^3 = 1$
$f(b) = f(2) = (2)^2 = 4$ $\quad$ $g(b) = g(2) = (2)^3 = 8$

Differentiate $f(x)$ & $g(x)$ with respect to x, we get

$f'(x) = 2x$ $\quad$ $g'(x) = 3x^2$ $\quad$ clearly $g'(x) \neq 0$ for any value
$f'(c) = 2c$ $\quad$ $g'(c) = 3c^2$ $\quad$ of x in $(1,2)$.

let $\dfrac{f'(c)}{g'(c)} = \dfrac{f(b) - f(a)}{g(b) - g(a)} \Rightarrow \dfrac{2c}{3c^2} = \dfrac{f(2) - f(1)}{g(2) - g(1)} \Rightarrow \dfrac{2}{3c} = \dfrac{4-1}{8-1}$

$$\Rightarrow \frac{2}{3c} = \frac{3}{7} \Rightarrow 9c = 14 \quad \therefore c = \frac{14}{9} = 1.5556$$

$\therefore c = 1.5556 \in (1,2)$ such that $\dfrac{f'\left(\frac{14}{9}\right)}{g'\left(\frac{14}{9}\right)} = \dfrac{3}{7} = \dfrac{f(2) - f(1)}{g(2) - g(1)}$

$\therefore$ Hence Cauchy's mean value theorem is verified.

Q54) Integrate the followings with respect x.

(a) x^{16}

$\rightarrow \int x^{16} dx$

we know;

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{16} dx = \frac{x^{16+1}}{16+1} + c$$

$$= \frac{x^{17}}{17} + c$$

(b) $\frac{1}{x^5}$

$\rightarrow \int \frac{1}{x} = \int x^{-5} dx$

we know

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$\int x^{-5} dx = \frac{x^{-5+1}}{-5+1} + c$$

$$= \frac{x^{-4}}{-4} + c$$

$$= \frac{-1}{4}(x^{-4}) + c$$

$$= \frac{-1}{4} \cdot \frac{1}{x^4} + c$$

$$= \frac{-1}{4x^4} + c$$

(c) $\sqrt[3]{x^4}$

$\rightarrow \int \sqrt[3]{x^4}\, dx$

$\therefore \frac{3}{7} x^{7/3} + c$

(d) $\frac{1}{e^{-x}}$

$\rightarrow \frac{1}{e^{-x}} \therefore e^x \quad \therefore \int e^x dx$

we know,

$$\int e^x dx = e^x + c$$

$$\therefore \int e^x dx = e^x + c$$

$$\therefore \boxed{e^x + c}$$

(e) $\frac{1}{\cos^2 x}$

$\rightarrow \int \frac{1}{\cos^2 x} dx = \int \sec^2 x\, dx$

$= \tan x + c$

$\frac{\tan x}{\cos x}$

$= \int \frac{\tan x}{\cos x} dx = \int \frac{1}{\cos x} \tan dx$

$= \int \sec x \tan x\, dx$

$= \sec x + c$

(f) $\frac{\tan x}{\cos x}$

$\longrightarrow$

$$\int \frac{\tan x}{\cos x}\,dx = \int \frac{1}{\cos x}\cdot \tan x\,dx$$

$$= \int \sec x \cdot \tan x\,dx$$

$$= \sec x + C.$$

(g) $\frac{\cos x}{\sin^2 x}$

$$\longrightarrow \int \frac{\cos x}{\sin^2 x}\,dx = \int \frac{1}{\sin x}\cdot\frac{\cos x}{\sin x}\,dx$$

$$= \int \operatorname{cosec} x \cot x$$

$$= -\operatorname{Cosec} + C$$

(h) $\frac{1}{25+x^2}$

$$\longrightarrow \int \frac{1}{25+x^2}\,dx$$

$$= \int \frac{1}{(5)^2+(x)^2}\,dx$$

We know,

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}(x/a)$$

$$\therefore \int \frac{1}{(5)^2+(x)^2}\,dx = \frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right)$$

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(I) $\frac{1}{\sqrt{36-x^2}}$

$\longrightarrow \int \frac{1}{\sqrt{36-x^2}} dx$

We know,

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{\sqrt{(6)^2-x^2}} dx = \sin^{-1}\left(\frac{x}{6^2}\right) + c$$

$$= \sin^{-1}\left(\frac{x}{6}\right) + c$$

(j) $\frac{1}{\sqrt{9+x^2}}$

$\longrightarrow \int \frac{1}{\sqrt{9+x^2}} dx$

We know,

$$\int \frac{1}{\sqrt{a^2+x^2}} dx = \log\left(x+\sqrt{a^2+x^2}\right) + c$$

$$\int \frac{1}{\sqrt{(3)^2+x^2}} dx = \log\left(x+\sqrt{3^2+x^2}\right) + c$$

$$= \log\left(x+\sqrt{9+x^2}\right) + c$$

(K) $e^x \sin 2x$ $\qquad a=1$ & $b=2$

$\longrightarrow \int e^x \sin 2x \, dx$

$$= \frac{e^{ax}}{a^2+b^2}\left(a \sin bx - b \cos bx\right) + c$$

$$= \frac{e^x}{5}\left(\sin 2x - 2\cos 2x\right) + c$$

Q55) Integrate the followings with respect x

(a) $(x+3)^5$

$\rightarrow \int (x+3)^5 dx$

We know,

$$\int (ax+b)^n dx = \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1} + c, (n \neq -1)$$

Here $a=1$, $b=3$ & $n=5$

$$\therefore \int (x+3)^5 dx = \frac{1}{1}\frac{(x+3)^{5+1}}{5+1} + c$$

$$\therefore \frac{(x+3)^6}{6} + c$$

(b) $(3x+4)^6$

$\rightarrow \int (3x+4)^6 dx$

We know,

$$\int (ax+b)^n dx = \frac{1}{a}\frac{(ax+b)}{n+1} + c, (n \neq -1)$$

$\therefore$ Here $a=3$, $b=4$ and $n=6$

$$\therefore \int (3x+4)^6 dx = \frac{1}{3}\frac{(3x+4)^{6+1}}{6+1} + c$$

$$\therefore \frac{1}{3}\frac{(3x+4)^7}{7} + c$$

$$\therefore \frac{(3x+4)^7}{21} + c$$

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(c) $(3-4x)^7$

$\longrightarrow \int (3-4x)^7 \, dx$

We know,

$$\int (ax+b)^n \, dx = \frac{1}{a} \frac{(ax+b)^{n+1}}{n+1} + C \quad \text{where } (n \neq -1)$$

$\therefore$ Here $a = -4$, $b = 3$ and $n = 7$

$$\therefore \int (3-4x)^7 \, dx = \frac{1}{-4} \frac{(3-4x)^{7+1}}{7+1} + C$$

$$= \frac{1}{-4} \frac{(3-4x)^8}{8} + C$$

$$\therefore \left(\frac{-1}{32}\right)(3-4x)^8 + C$$

(d) e^{3x+2}

$\longrightarrow \int (e^{3x+2}) \, dx$.

We know

$$\int e^{ax+b} \, dx = \frac{1}{a} e^{ax+b} + C$$

Here $a = 3$, $b = 2$

$$\therefore \int e^{3x+2} \, dx = \frac{1}{3} e^{3x+2} + C$$

$$\therefore \frac{1}{3} e^{3x+2} + C$$

(e) e^{8-4x}

$\rightarrow \int e^{8-4x} dx$

We know,

$$\int e^{ax-b} dx = \frac{1}{a} e^{ax+b} + C$$

Here $a = -4$ and $b = 8$

$$\therefore \int e^{8-4x} dx = \frac{1}{-4} e^{8-4x} + C$$

$$\therefore \left(\frac{-1}{4}\right) e^{8-4x} + C$$

(f) $\frac{1}{3x+2}$

$\rightarrow \int \frac{1}{3x+2} dx$

We know,

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \log(ax+b) + C$$

Here $a = 3$ and $b = 2$

$$\therefore \int \frac{1}{3x+2} dx = \frac{1}{3} \log(3x+2) + C$$

$$\therefore \frac{1}{3} \log(3x+2) + C$$

(g) $\frac{1}{(2x+3)^5}$

$\rightarrow \int \frac{1}{(2x+3)^5} dx = \int (2x+3)^{-5} dx$

We know

$$\int (ax+b)^n dx = \frac{1}{a} \frac{(ax+b)^{n+1}}{n+1} + c \quad \text{where, } (n \neq -1)$$

Here $a=2$, $b=3$ and $n=-5$

$$\therefore \int (2x+3)^{-5} dx = \left(\frac{1}{2}\right) \frac{(2x+3)^{-5+1}}{-5+1} + c$$

$$= \left(\frac{1}{2}\right) \frac{(2x+3)^{-4}}{-4} + c$$

$$= \frac{1}{-8} \frac{1}{(2x+3)^4} + c$$

$$\therefore \left(\frac{-1}{8}\right) \frac{1}{(2x+3)^4} + c$$

(h) $\sin(2x+4)$

$\rightarrow \int \sin(2x+4) dx$

We know:

$$\int \sin(ax+b) dx = \frac{-1}{a} \cos(ax+b) + c$$

Here $a=2$, $b=4$

$$\therefore \int \sin(2x+4) dx = \frac{-1}{2} \cos(2x+4) + c$$

$$= \left(\frac{-1}{2}\right) \cos(2x+4) + c$$

Excellent

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Notes

provided

by

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