★ Number System

Numbers :- There are 7 type of Numbers.

- Natural Numbers :-
  Natural Numbers are the numbers that start from 1 and ends at infinity.
  ∴ These are denoted by [N]
  N = 1, 2, 3, 4, 5.

- Whole Numbers :- Whole Numbers are positive number including 0, without any decimal or fractional parts. They are numbers that represent whole things without pecies.
  These are denoted by [W]
  W = 0, 1, 2, 3, 4.

- Integer :-
  An integer is any number including 0, positive number and negative numbers. It should be noted that an integer can never be a fraction decimal or a present.

These are denoted by [Q]
$Q = \frac{2}{3}, \frac{4}{5}$ but not $\frac{3}{0}, \frac{2}{0}$ } example of Rational Number.

- Irrational Numbers :- An irrational numbers is a real number that cannot be expressed as a ratio of integer. The number consisting of radicals / roots are called Irrational number.

These are denoted by $[Q^c]$

$Q = \frac{2}{3}, \frac{4}{5}$ but $\frac{3}{0}, \frac{2}{0}$.

$Q^c = \sqrt{5}, \sqrt{3}, \sqrt{2}$ etc

- Rational Number :-

A rational Number is a Number that can be written as a fraction where the numerator and denominator are both integers and the denominater not 0

Numbers that are in the form of P/q where $q \neq 0$ are called rational numbers

These are denoted by [Q]

$Q = \frac{2}{3}, \frac{4}{5}$ but not $\frac{3}{0}, \frac{2}{0}$.

- Real Number :

Real Number include rational numbers like positive or negative integers, fraction and irrational numbers.

In other words any number that we can think of except complex number is a real number

They are denoted by [R]

$R = \{Q\ Q^c\}$ or $3, 0, \sqrt{5}, 1.5, 3/2$ etc.

- Complex Numbers :-

Complex Numbers are the numbers that are expressed in the form of $a+ib$ where a, b are real Numbers and i is an imagnery number.

These are denoted by $[c = a+ib]$

ex :- $3 + i6$.

Matrices

Matrices :- 1.1 [date - 3/9/24]

Types of Matrices — 1.2

Elementary properties of Matrices — 1.3

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Symmetric — 1.6

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1.1 Matrix :-

Matrix is a set of numbers arrange in row and columns so as to form a rectangular array. The numbers are called the elements, entries of the matrix, each element of a matrix is obtain denoted by a variable with two subscripts.

For ex :- $a_{2,1}$ represents the elements at the second row and first column of the matrix.

$$A = \begin{bmatrix} a_{11}, & a_{12}, & a_{13} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots & a_{2,n} \\ \vdots & & & & \\ a_{m,1} & a_{m,2}, & a_{m,3} & \cdots & a_{m,n} \end{bmatrix}$$

* Uses of Matrixes :- Matrices are generally used in solving simuntaneous equations are linear transformation. The theory of Matrices is also apply in differential equations, astronomy, mechanics, theory of electric circuit etc.

* Various types of Matrices :-

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1. Row Matrix :- If a matrix has only one row and any number of columns it is called a row matrix.

eg :- $[2, 3, 7, 9]$

2. Column Matrix :- A matrix having one column and any number of rows is called column matrix.

Eg :- $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

3. Null Matrix Or Zero Matrix :- Any matrix in which all elements are zero's is called Null matrix or Zero matrix.

eg :- $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

4. Square Matrix :- A Matrix in which the numbers of columns is called Square matrix.

Eg :- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$. $\begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix}$

5. Diagonal Matrix :- A square matrix is called a diagonal matrix if its a non-diagonal elements are zero.

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eg $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

6.) Scalar Matrix :- A diagonal matrix in which all the diagonal elements are equal to a Scaler say (k) is called a Scaler matrix.

eg:- $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ $\begin{bmatrix} -6 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 \\ 0 & 0 & -6 & 0 \end{bmatrix}$

i.e = $A = [a_{ij}]_{mn}$ is a scaler matrix

$a_{ij} = 0$ when $i \neq j$

k when $i = j$.

7.) Identity / unit Matrix :- A Square matrix is called a unit Matrix if all diagonal elements are unity and non diagonal elements are 0.

eg:- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

8.] Symmetric Matrix : A square matrix will be called Symmetric if for all values of i and j

eg: $a_{ij} = a_{ij}$

i.e. $A' = A$

eg: $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

9.] Skew Symmetric Matrix :- A square Matrix is called skew symmetric matrix Its if

1.) $a_{ij} = -a_{ij}$ for all values of i and j

or. $A' = -A$

2.) All diagonal elements are zero.

eg:- $\begin{bmatrix} 0 & -h & -g \\ h & 0 & -f \\ g & f & 0 \end{bmatrix}$

10.) Triangular Matrix :- A square Matrix is all of those elements below the leading diagonal are zero is called an upper Triangular Matrix. A square matrix of whose elements above the leading diagonal are zero is called a lower triangular matrix.

eg :- $\begin{bmatrix} 1 & 3 & 2 \\ 0 & 4 & 1 \\ 0 & 0 & 6 \end{bmatrix}$ upper Triangular Matrix.

$\begin{bmatrix} 2 & 0 & 0 \\ 4 & 1 & 0 \\ 5 & 6 & 7 \end{bmatrix}$ Lower triangular Matrix

11) Transpose matrix :- If in a given matrix A we interchange the rows and the corresponding columns. The New matrix obtained is called the matrix A'. A' is the new which is denoted by A' or AT.

eg :- $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 5 \\ 6 & 7 & 8 \end{bmatrix}$, $A' = \begin{bmatrix} 2 & 1 & 6 \\ 3 & 0 & 7 \\ 4 & 5 & 8 \end{bmatrix}$

12) orthogonal matrix :- A square matrix A is called an orthogonal matrix if the product of the matrix A and the tranpose matrix A is an identity matrix

$A \cdot A' = I$ If $|A| = 1$, A is proper.

13) Conjugate of matrix :-

Let $A = \begin{bmatrix} 1+i & 2-3i & 4 \\ 7+2i & -i & 3-2i \end{bmatrix}$

Conjugate of matrix A is $\bar{A}$

$$\bar{A} = \begin{bmatrix} 1-i & 2+3i & 4 \\ 7-2i & i & 3+2i \end{bmatrix}$$

14) Matrix A^{θ} Transpose of the conjugate of a matrix A is denoted by A^{θ}.

$$A = \begin{bmatrix} 1+i & 2-3i & 4 \\ 7+2i & -i & 3-2i \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 1-i & 2+3i & 4 \\ 7-2i & +i & 3+2i \end{bmatrix}$$

$$(\bar{A})' = \begin{bmatrix} 1-i & 7-2i \\ 2+3i & i \\ 4 & 3+2i \end{bmatrix}$$

$$A^{\theta} = \begin{bmatrix} 1-i & 7-2i \\ 2+3i & i \\ 4 & 3+2i \end{bmatrix}$$

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Unitary Matrix : A square matrix A is said to be unitary if

$$A^{\theta} A = I$$

$$A = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix} \quad A^{\theta} = \begin{bmatrix} \frac{1-i}{2} & \frac{1-i}{2} \\ \frac{-1-i}{2} & \frac{1+i}{2} \end{bmatrix}$$

$$A \cdot A^{\theta} = 1.$$

Hermitian Matrix :- A square matrix $A = (a_{ij})$ is called Hermitian matrix. If every i-j^{th} element of A is equal to conjugate complex j-ith element of A.

In other words $a_{ij} = \bar{a}_{ji}$

$$\begin{bmatrix} 1 & 2+3i & 3+i \\ 2-3i & 2 & 1-2i \\ 3-i & 1+2i & 5 \end{bmatrix}$$

Necessary and sufficient condition for a matrix A to be Hermitian is that $A = A^{\theta}$ i.e conjugate transpose of A

$$A = (\bar{A})'$$

* Skew Hermitian Matrix :- A Square matrix $A = (a_{ij})$ will be called a Skew Hermitian matrix if every i-jth element of A is equal to negative complex of j-ith element A.

In other words, $a_{ij} = -\bar{a}_{ij}$.

All the elements in the principal diagonal will be of the form

If $a_{ij} = -\bar{a}_{ij}$ or $a_{ij} + \bar{a}_{ij} = 0$
$a_{ij} = a + ib$ then $\bar{a}_{ij} = a - ib$

$(a+ib) + (a-ib) = 0 \Rightarrow 2a = 0$
$\boxed{a = 0}$

* Idempotent matrix :- A matrix, Such that $A^2 = A$ is called Idempotent matrix.

eg: $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ $A^2 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = A$.

* Nilpotent matrix :- A matrix will be called a Nilpotent if $A^K = 0$ (null matrix) where k is +ve integer; if however K is the least +ve

integer for $A^k = 0$ then k is the index of the nilpotent matrix.

eg: $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ $A^2 = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

A is nilpotent matrix whose index is 2.

* Involuntary Matrix :- A will Matrix 'A' will be called an Involunatry matrix, if $A^2 = I$ (unit matrix). Since $I^2 = I$ always ∴ unit matrix is involuntary.

* Equal Matrices :- Two matrices are said to be equal if

(i) They are of the same order.
(ii) The elements in the corresponding positions are equal.

Thus if $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

Here ;

(w) Singular matrix eg $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ is singular matrix because $|A| = 6-6 = 0$

Multiplication :-

The product of two matrices A and B is only possible if the number of columns in A is equal to the number of rows in B.
Let $A = [a_{ij}]$ be an $m \times n$ matrix and $B = [b_{ij}]$ be an $n \times p$ matrix. Then the product AB of these matrices is an $m \times p$ matrix $C = [c_{ij}]$ where

$$c_{ij} = a_{i1} \cdot b_{1j} + a_{i2}\, b_{2j} + a_{i3}\, b_{3j} + \dots + a_{in}\, b_{nj}.$$

Examples :-

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1\times1 + 1\times0 + 1\times1 & -2\times1 + -2\times0 + -2\times1 & 3\times1 + 3\times0 + 3\times1 \\ 2\times0 + 2\times1 + 2\times2 & 3\times0 + 3\times1 + 3\times2 & -1\times0 + -1\times1 + -1\times2 \\ -3\times2 + -3\times2 + -3\times0 & 1\times2 + 1\times2 + 1\times0 & 2\times2 + 2\times2 + 2\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+1 & -2+0+-2 & 3+0+3 \\ 0+2+4 & 0+3+6 & 0+-1+-2 \\ -6+-6+0 & 2+2+0 & 4+4+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -4 & 6 \\ 6 & 9 & -3 \\ -12 & 4 & 8 \end{bmatrix}$$

Ex 2.

$$A = \begin{bmatrix} 2 & -4 & 6 \\ 6 & 9 & -3 \\ -12 & 4 & 8 \end{bmatrix} \qquad B = \begin{bmatrix} -6 & 8 & -7 \\ 3 & 4 & 4 \\ 2 & -3 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2\times-6 \quad 2\times8 \quad 2\times-7 & -4\times3 \quad -4\times4 \quad -4\times4 & 6\times2 \quad 6\times-3 \quad 6\times \\ 6\times-6 \quad 6\times8 \quad 6\times-7 & 9\times3 \quad 9\times4 \quad 9\times4 & -3\times2 \quad -3\times-3 \quad -3\times \\ -12\times-6 \quad -12\times8 \quad -12\times-7 & 4\times3 \quad 4\times4 \quad 4\times4 & 8\times2 \quad 8\times-3 \quad 8\times \end{bmatrix}$$

$$AB = \begin{bmatrix} (-12)+16+(-14) & (-12)+(-16)+16 & 12+(-18)+12 \\ (-36)+48+(-42) & 27+36+36 & (-6)+9+(-6) \\ 72+(-96)+(+84) & 12+16+16 & 16+(-24) \quad 16 \end{bmatrix}$$

$$AB = \begin{bmatrix} -10 & -12 & -6 \\ 30 & 99 & 21 \\ -18 & 44 & -3 \end{bmatrix}$$

(3) $$A = \begin{bmatrix} 2 & 3 & 9 \\ 4 & 2 & 4 \\ 6 & 1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2\times2+2\times3+2\times4 & 3\times2+3\times3+3\times4 & 9\times2+9\times3+9\times4 \\ 4\times2+4\times3+4\times4 & 2\times2+2\times3+2\times4 & 4\times2+4\times3+4\times4 \\ 6\times2+6\times3+6\times4 & 1\times2+1\times3+1\times4 & 2\times2+2\times3+2\times4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4+6+8 & 6+9+12 & 18+27+36 \\ 8+12+16 & 4+6+8 & 8+12+16 \\ 12+18+24 & 2+3+4 & 4+6+8 \end{bmatrix}$$

$$AB = \begin{bmatrix} 18 & 27 & 81 \\ 36 & 18 & 36 \\ 54 & 9 & 18 \end{bmatrix}$$

4] $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 6 & 3 \\ 6 & 3 & 6 \end{bmatrix} \quad \begin{bmatrix} -2 & 1 & -4 \\ 3 & -6 & 3 \\ -6 & 3 & -6 \end{bmatrix}$

$$AB = \begin{bmatrix} 1\times-2 + 1\times3 + 1\times-6 & 2\times-2 + 2\times3 + 2\times-6 & 4\times-2 + 4\times3 + 4\times-6 \\ 3\times1 + 3\times-6 + 3\times3 & 6\times3 + 6\times-6 + 6\times3 & 3\times1 + 3\times-6 + 3\times3 \\ 6\times-4 + 6\times3 + 6\times-6 & 3\times-4 + 3\times3 + 3\times-6 & 6\times-4 + 6\times3 + 6\times-6 \end{bmatrix}$$

$$AB = \begin{bmatrix} -2+3-6 & -4+6-12 & -8+12-24 \\ 3+8+9 & 6\cancel{-4}-36+12 & 3-18+9 \\ -24+18+36 & -12+9-18 & -24+18-36 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -10 & -20 \\ -6 & -12 & -6 \\ -42 & -21 & -42 \end{bmatrix}$$

* **Matrix Addition** :- 2 Matrices A and B can be added if

(1) order is same

(2) corresponding elements will be added.

eg: $A = \begin{bmatrix} 3 & 2 & -5 \\ -2 & 4 & 7 \\ 5 & -7 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 7 & -11 & -13 \\ 11 & 8 & 12 \\ 15 & -12 & 9 \end{bmatrix}$

$A+B = \begin{bmatrix} 3+7+3+11+3+15 & 2+7+2+11+2+5 \\ -2+(-11)+(-2)+8+(-2)+(-12 & 4(-11)+4+8+4+(-12) \\ 5+(-15)+5+12+5+9 & -7+(-15)+(-7)+12\ (-7+9) \end{bmatrix}$

$\begin{bmatrix} (-5+7)+(-5)+11+(-5+15) \\ 7+(-11)+7+8+7+(-2) \\ \cancel{5+(-15)+(-5)+(-12)+(-5)+9} \\ 5+(-15)+5+12+5+9 \end{bmatrix}$

$= \begin{bmatrix} 10+14+18 & 9+3+17 & 2+6+10 \\ (-13)+6+(-14) & -7+12+(-8) & (-4)+15+(-5) \\ (-10)+17+14 & -22+5+2 & (-10)+17+14 \end{bmatrix}$

$= \begin{bmatrix} 42 & 39 & 18 \\ -21 & -3 & 6 \\ 21 & -15 & 21 \end{bmatrix}$

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* ASSIGNMENT

Addtion Matrix

1.) $\begin{bmatrix} 1 & 4 \\ 5 & -2 \end{bmatrix} + \begin{bmatrix} 7 & 3 \\ 11 & 9 \end{bmatrix}$

$\begin{bmatrix} 8 & 7 \\ 16 & 7 \end{bmatrix}$

2.] $\begin{bmatrix} -3 & 4 \\ 6 & 13 \end{bmatrix} + \begin{bmatrix} 3 & -8 \\ -2 & 2 \end{bmatrix}$

$\begin{bmatrix} 0 & -4 \\ 4 & 15 \end{bmatrix}$

3.] $\begin{bmatrix} 5 & 7 & 9 \\ 3 & -2 & 2 \\ 1 & 0 & 7 \end{bmatrix} + \begin{bmatrix} -2 & 5 & -6 \\ 4 & -3 & 5 \\ -1 & \sqrt{3} & 8 \end{bmatrix}$

$\begin{bmatrix} 3 & 12 & 3 \\ 7 & -5 & 7 \\ 0 & \sqrt{3} & 15 \end{bmatrix}$

4.) $\begin{bmatrix} -7 & 5 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} -14 & 3 \\ -3 & 1 \end{bmatrix}$

$\begin{bmatrix} -21 & 8 \\ 1 & 4 \end{bmatrix}$

5.) $\begin{bmatrix} 1 & 6 & 4 \\ -3 & 8 & 9 \\ -2 & 7 & -1 \end{bmatrix} + \begin{bmatrix} -2 & 4 & -1 \\ 4 & -17 & -5 \\ 6 & 1 & 8 \end{bmatrix}$

$\begin{bmatrix} -1 & 10 & 3 \\ 1 & -9 & 4 \\ 4 & 8 & 7 \end{bmatrix}$

6.] $\begin{bmatrix} -8 & -6 \\ -12 & 33 \end{bmatrix} + \begin{bmatrix} 9 & -43 \\ 34 & -17 \end{bmatrix}$

$\begin{bmatrix} 1 & -49 \\ 22 & 16 \end{bmatrix}$

7.] $\begin{bmatrix} -16 \\ 38 \end{bmatrix} + \begin{bmatrix} 27 \\ -13 \end{bmatrix}$

$\begin{bmatrix} \cancel{29} \\ \cancel{-26} \\ \cancel{-8} \end{bmatrix} \begin{bmatrix} 11 \\ 25 \end{bmatrix}$

8.] $\begin{bmatrix} 66 & -24 & -18 \end{bmatrix} + \begin{bmatrix} -48 & 28 & -28 \end{bmatrix}$

$\begin{bmatrix} 18 & 4 & -46 \end{bmatrix}$

9.] $\begin{bmatrix} 17 \\ -49 \\ -5 \end{bmatrix} + \begin{bmatrix} 12 \\ 23 \\ -3 \end{bmatrix}$

$$\begin{bmatrix} 29 \\ -26 \\ -8 \end{bmatrix}$$

10.] $\begin{bmatrix} 43 & 17 \\ -27 & 28 \\ 18 & 37 \end{bmatrix} + \begin{bmatrix} 2 & 18 \\ 16 & 3 \\ -11 & -6 \end{bmatrix}$

$$\begin{bmatrix} 45 & 35 \\ -11 & 31 \\ 7 & 31 \end{bmatrix}$$

11.] $\begin{bmatrix} 3 & 12 \\ -11 & 2 \end{bmatrix} + \begin{bmatrix} 17 & -3 \\ 25 & 17 \end{bmatrix} + \begin{bmatrix} -35 & 21 \\ 8 & -4 \end{bmatrix}$

$$\begin{bmatrix} 3+17+(-35) & 12+(-3)+21 \\ (-11)+25+8 & 2+17+(-4 \end{bmatrix}$$

$$\begin{bmatrix} -15 & 30 \\ 22 & 18 \end{bmatrix}$$

12.] $\begin{bmatrix} 21 & 3 \\ -7 & 12 \\ -3 & 4 \end{bmatrix} + \begin{bmatrix} 11 & 32 \\ -15 & 3 \\ 8 & 6 \end{bmatrix} + \begin{bmatrix} 3 & -7 \\ 23 & 5 \\ 5 & 2 \end{bmatrix}$

$$\begin{bmatrix} 21+11+3 & 3+32+(-7) \\ -7+(-15)+23 & 12+3+5 \\ (-3)+8+5 & 4+6+2 \end{bmatrix}$$

$$= \begin{bmatrix} 35 & 28 \\ 1 & 20 \\ 10 & 12 \end{bmatrix}$$

13.] $\begin{bmatrix} 8 & 2 & 3 \\ 2 & -7 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 7 \\ 5 & 8 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 1 & 2 \\ 6 & 8 & 7 \end{bmatrix}$

$$\begin{bmatrix} 8+3+4 & 2+4+1 & 3+7+2 \\ 2+5+6 & -7+8+8 & 2+3+7 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 7 & 12 \\ 13 & 9 & 12 \end{bmatrix}$$

14.] $\begin{bmatrix} -12 & 0 \\ 5 & 0 \end{bmatrix} + \begin{bmatrix} 17 & 12 \\ 38 & 13 \end{bmatrix} + \begin{bmatrix} 10 & -5 \\ -25 & -5 \end{bmatrix}$

$$\begin{bmatrix} -12+17+10 & 0+12+(-5) \\ 5+38+(-25) & 0+13+(-5) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 7 \\ 18 & 8 \end{bmatrix}$$

15.] $\begin{bmatrix} -8 & 12 & -8 \\ 6 & 5 & -7 \\ 7 & 10 & -7 \end{bmatrix} + \begin{bmatrix} -12 & 8 & -6 \\ 6 & 0 & 4 \\ -5 & 5 & 10 \end{bmatrix} + \begin{bmatrix} 2 & 0 & -1 \\ -5 & 5 & 4 \\ 3 & -8 & 6 \end{bmatrix}$

$$\begin{bmatrix} (-8)+(-12)+2 & 12+8+0 & (-8)+(-6)+(-1) \\ 6+0+(-5) & 5+0+5 & (-7)+4+4 \\ 7+(-5)+3 & 10+5+(-8) & (-7)+10+6 \end{bmatrix}$$

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$$= \begin{bmatrix} -18 & 20 & -15 \\ 1 & 10 & 11 \\ 5 & 7 & 9 \end{bmatrix}$$

16.] $\begin{bmatrix} -2 & 2 & 3 \\ -2 & 5 & -4 \end{bmatrix} + \begin{bmatrix} -2 & 6 & -3 \\ 2 & -1 & -5 \end{bmatrix}$

$$\begin{bmatrix} -4 & 8 & 0 \\ 0 & 4 & -9 \end{bmatrix}$$

17 $\begin{bmatrix} -2 & 4 & -4 \\ 3 & 4 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 3 & 0 \\ -6 & 4 & 4 \end{bmatrix}$

$$\begin{bmatrix} -2 & 7 & -4 \\ -3 & 8 & 3 \end{bmatrix}$$

18.) $\begin{bmatrix} 5 \\ -1 \\ 6 \\ -6 \end{bmatrix} - \begin{bmatrix} -1 \\ -5 \\ -2 \\ -3 \end{bmatrix}$

$$\begin{bmatrix} 6 \\ 4 \\ 8 \\ -3 \end{bmatrix}$$

19] $\begin{bmatrix} 2 & 4 & -1 & -2 \\ -1 & -3 & -6 & -2 \end{bmatrix} \begin{bmatrix} 0 & 3 & -6 & 0 \\ 3 & -1 & 2 & -3 \end{bmatrix}$

$$\begin{bmatrix} 2 & 1 & -5 & -2 \\ -4 & -2 & -8 & 1 \end{bmatrix}$$

20.] $\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 2 & 5 \end{bmatrix}$

$\begin{bmatrix} 7 & 4 & 6 \end{bmatrix}$

21.] $\begin{bmatrix} 1 & -6 \\ -5 & 0 \\ -5 & 4 \end{bmatrix} - \begin{bmatrix} -3 & -3 \\ -5 & -1 \\ -3 & -6 \end{bmatrix}$

$\begin{bmatrix} 4 & -3 \\ 0 & 1 \\ -2 & 10 \end{bmatrix}$

22.] $\begin{bmatrix} -5 & 1 & 1 \\ 5 & 5 & -4 \end{bmatrix} - \begin{bmatrix} 5 & 5 & 2 \\ 0 & -4 & 3 \end{bmatrix}$

$\begin{bmatrix} -10 & -4 & -1 \\ 5 & 9 & -7 \end{bmatrix}$

23.) $\begin{bmatrix} -5 \\ 3 \\ 6 \end{bmatrix} - \begin{bmatrix} -1 \\ -5 \\ 6 \end{bmatrix}$

$\begin{bmatrix} -4 \\ 8 \\ 0 \end{bmatrix}$

24.] $\begin{bmatrix} -1 & 6 \\ 5 & 2 \\ -5 & -1 \\ 4 & -4 \end{bmatrix} - \begin{bmatrix} -3 & 5 \\ 4 & 1 \\ 1 & -1 \\ 4 & -4 \end{bmatrix}$

$$\begin{bmatrix} -4 & 11 \\ 9 & 3 \\ -4 & -2 \\ 8 & -8 \end{bmatrix}$$

25 $\begin{bmatrix} 4 & -2 & 3 & 4 \end{bmatrix} + \begin{bmatrix} -6 & 2 & -3 & 4 \end{bmatrix}$

$\begin{bmatrix} -2 & 0 & 0 & 8 \end{bmatrix}$

26 $\begin{bmatrix} 0 & -5 & -3 & -4 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 4 & -1 \end{bmatrix}$

$\begin{bmatrix} -1 & -5 & -7 & -3 \end{bmatrix}$

27.] $\begin{bmatrix} 5 & 5 & -2 & 2 \\ -1 & 6 & 5 & -6 \end{bmatrix} - \begin{bmatrix} 1 & 1 & -2 & -2 \\ -1 & 3 & -2 & -6 \end{bmatrix}$

$\begin{bmatrix} 4 & 4 & 0 & 4 \\ 0 & 3 & 7 & 0 \end{bmatrix}$

28.] $\begin{bmatrix} 5 & -2 & -1 & -1 \\ -3 & -5 & 6 & 2 \end{bmatrix} + \begin{bmatrix} -6 & 6 & -5 & 4 \\ 0 & 2 & 2 & 4 \end{bmatrix}$

$\begin{bmatrix} -1 & 4 & -6 & 3 \\ -3 & -3 & 8 & 6 \end{bmatrix}$

29.] $\begin{bmatrix} 5 & -4 & 6 & -1 \end{bmatrix} + \begin{bmatrix} -3 & -1 & -2 & -3 \end{bmatrix}$

$\begin{bmatrix} 2 & -5 & 4 & -4 \end{bmatrix}$

30.] $\begin{bmatrix} -3 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} -5 \\ -2 \\ -3 \end{bmatrix} = \begin{bmatrix} -8 \\ 1 \\ -3 \end{bmatrix}$

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31 $\begin{bmatrix} 3 & -3 & 4 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 2 & -3 & 4 \end{bmatrix}$

$\begin{bmatrix} 7 & -5 & 7 & -4 \end{bmatrix}$

32) $\begin{bmatrix} -6 & -3 \\ -1 & 3 \\ -4 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 6 \\ 3 & 6 \end{bmatrix}$

$\begin{bmatrix} -5 & 3 \\ 4 & 9 \\ -1 & 5 \end{bmatrix}$

33) $\begin{bmatrix} 5 & 3 \\ -5 & -1 \\ 0 & 0 \\ -4 & 3 \end{bmatrix} + \begin{bmatrix} -2 & 5 \\ -5 & 1 \\ -6 & -6 \\ 5 & -6 \end{bmatrix}$

$\begin{bmatrix} 3 & 8 \\ -10 & 0 \\ -6 & -6 \\ 1 & -3 \end{bmatrix}$

34) $\begin{bmatrix} -6 & -2 \\ -2 & -1 \\ -6 & 5 \end{bmatrix} + \begin{bmatrix} 4 & -5 \\ 4 & -4 \\ -6 & -5 \end{bmatrix}$

$\begin{bmatrix} -2 & 7 \\ 2 & -5 \\ -12 & 0 \end{bmatrix}$

$$\begin{bmatrix} 5 & 7 & 7 & 1 \end{bmatrix}$$

★ # MATRIX MULTIPLICATION :-

36.] $\begin{bmatrix} 0 & 2 \\ -2 & -5 \end{bmatrix} \cdot \begin{bmatrix} 6 & -6 \\ 3 & 0 \end{bmatrix}$

$$\begin{bmatrix} 0\times 6 + 0\times 3 & 2\times 6 + 2\times 3 \\ (-2)\times(-6) + (-2)\times 0 & (-5)\times(-6) + (-5)\times 0 \end{bmatrix}$$

$$\begin{bmatrix} 0+0 & 12+6 \\ 12+0 & -30+0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 18 \\ 12 & -30 \end{bmatrix}$$

37.) $\begin{bmatrix} 6 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} -5 & 4 \end{bmatrix}$

$$\begin{bmatrix} 6\times -5 & 6\times 4 \\ -3\times -5 & -3\times 4 \end{bmatrix}$$

$$\begin{bmatrix} -30 & 24 \\ 15 & -12 \end{bmatrix}$$

38.) $\begin{bmatrix} -5 & -5 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} -2 & -3 \\ 3 & 5 \end{bmatrix}$

$\begin{bmatrix} (-5)\times(-2) + (-5)\times(3) & (-5)\times(-2) + (-5)\times(3) \\ (-1)\times 3 + (-1)\times 5 & 2\times(-3) + 2\times 5 \end{bmatrix}$

$\begin{bmatrix} 10 + (-15) & 10 + (-15) \\ -3 + (-5) & -6 + 10 \end{bmatrix}$

$\begin{bmatrix} -5 & -5 \\ -8 & 4 \end{bmatrix}$

39.) $\begin{bmatrix} -3 & 5 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 6 & -2 \\ 1 & -5 \end{bmatrix}$

$\begin{bmatrix} -3\times 6 + -3\times 1 & 5\times 6 + 5\times 1 \\ (-2)\times(-2) + (-2)\times(-5) & 1\times(2) + 1\times(-5) \end{bmatrix}$

$\begin{bmatrix} -18 + (-3) & 30 + 5 \\ 4 + 10 & -2 + (-5) \end{bmatrix}$

$\begin{bmatrix} -21 & 35 \\ 14 & -7 \end{bmatrix}$

40.) $\begin{bmatrix} 0 & 5 \\ -3 & 1 \\ -5 & 1 \end{bmatrix} \cdot \begin{bmatrix} -4 & 4 \\ -2 & -4 \end{bmatrix}$

$\begin{bmatrix} (-4)\times 0 + (-4)\times(-3) + (-4)\times(-5) & 4\times 0 + 4\times(-3) + 4\times(-5) \\ (-2)\times 5 + (-2)\times 1 + (-2)\times 1 & (-4)\times 5 + (-4)\times 1 + (-4)\times 1 \end{bmatrix}$

$$\begin{bmatrix} 0+12+20 = & 0+(-12)+(-20) \\ -10+(-2)+(-2) - & (-20)+(-4)+(-4) \end{bmatrix}$$

$$\begin{bmatrix} 32 & -32 \\ -14 & -28 \end{bmatrix}$$

41)
$$\begin{bmatrix} 5 & 3 & 5 \\ 1 & 5 & 0 \end{bmatrix} \quad \begin{bmatrix} -4 & 2 \\ -3 & 4 \\ 3 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 5\times(-4)+5\times(-3)+5\times3 & 3\times(-4)+3(-3)+(3\times3) & 5\times(-4)+5\times-3+5\times3 \\ 1\times2+1\times4+1\times(-5) & 5\times2+5\times4+5\times(-5) & \cancel{0\times}2+0\times4+0\times(-5) \end{bmatrix}$$

$$\begin{bmatrix} -20+(-15)+15 & (-12)+(-9)+9 & -20+(-15)+15 \\ 2+4+(-5) & 10+20+(-25 & 0+0+0 \end{bmatrix}$$

$$\begin{bmatrix} -20 & -12 & -20 \\ 1 & 5 & 0 \end{bmatrix}$$

42)
$$\begin{bmatrix} -5 \\ 6 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \end{bmatrix}$$

$$\begin{bmatrix} (-5)\times3 & -5\times(-1) \\ 6\times3 & 6\times(-1) \\ 0\times3 & 0\times(-1) \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ 18 & -6 \\ 0 & 0 \end{bmatrix}$$

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$$\begin{bmatrix} 5 & -4 & -2 \\ 5 & -5 & 4 \\ 2 & 5 & -4 \\ -5 & 4 & 3 \\ 3 & -4 & -3 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 5\times5 + (-4)\times(-2) + (-2\times(-3)) \\ 5\times5 + (-5)\times(-2) + 4\times(-3) \\ 2\times5 + (5)\times(-2) + (-4)\times(-3) \\ (-5)\times5 + (4)(2) + 3(-3) \\ 3(5) + (-4)(-2) + (-3)(-3) \end{bmatrix}$$

$$\begin{bmatrix} 25 + 8 + 6 \\ 25 + 10 + (-12) \\ 10 + (-10) + 12 \\ 25 + (-8) + (-9) \\ 15 + 8 + 9 \end{bmatrix}$$

$$\begin{bmatrix} 39 \\ 23 \\ 12 \\ -49 \\ 32 \end{bmatrix}$$

44) $\begin{bmatrix} 4 & -1 \end{bmatrix} \cdot \begin{bmatrix} 4 & -4 \\ 5 & 5 \end{bmatrix}$

$$\begin{bmatrix} 4\times4 + (-1)\times5 & 4\times(-4) + (-1)\times5 \end{bmatrix}$$

$$\begin{bmatrix} 11 & -21 \end{bmatrix}$$

45]

$$\begin{bmatrix} 2 \\ 5 \\ -3 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} -2 & -5 & -4 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 2\times(-2) & 2\times(-5) & 2\times(-4) & 2\times(-1) \\ 5\times(-2) & 5\times(-5) & 5\times(-4) & 5\times(-1) \\ (-3)\times(-2) & (-3\times -5) & (-3\times -4) & (-3\times -1) \\ (-2)\times(-2) & (-2)\times(-5) & (-2\times -4) & (-2\times -1) \end{bmatrix}$$

$$\begin{bmatrix} -4 & -10 & -8 & -2 \\ -10 & -25 & -20 & -5 \\ 6 & 15 & 12 & 3 \\ 4 & 10 & 8 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -1 \end{bmatrix} \cdot \begin{bmatrix} -3 & 4 & -4 & 4 \\ 0 & 4 & -5 & -1 \end{bmatrix}$$

$-8 + (-4)$

$8 + 1$

$[(-2)\times(-3) + (-2)\times 0 \quad (-2)\times 4 + (-1)\times 4 \quad (-2)\times(-4) + (-1)\times(-5)$

$(-2)\times 4 + (-1)\times 1]$

$[(-1)(-3) + (-1)(0) + (-1)(4) + (-1)(4) + (-1)(-4) + (-1)(-5) \quad (-1)(4) + (-1)(-1)$

$[\cancel{(-2\times -3)}$ $\begin{bmatrix} 6 & -12 & 13 & -9 \end{bmatrix}$

$(-2)\times(-3) + (-1)\times 0$

47.)

$$\begin{bmatrix} -3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -4 & 3 & 3 & -2 & 3 \\ -1 & -4 & -5 & -5 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -3\times 5 + 4\times -1 & -3\times -4 + 4\times -4 & -3\times 3 + 4\times -5 \\ -3\times(-2) + 4\times 1 & -3\times 3 + 4\times 2 \end{bmatrix}$$

$$\begin{bmatrix} (-15)+(-4) & 12 + -16 & -9 + (-20) & 6+4 & (-9)+8 \end{bmatrix}$$

$$\begin{bmatrix} -19 & -4 & -29 & 10 & -1 \end{bmatrix}$$

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$$\begin{bmatrix} 4 & -2 \end{bmatrix} \begin{bmatrix} -5 & 5 \\ -5 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 4\times(-5) + (-2)\times(-5) & 4\times 5 + (-2)\times(-4) \end{bmatrix}$$

$$\begin{bmatrix} -20 + 10 & 20 + 8 \end{bmatrix}$$

$$\begin{bmatrix} -10 & 28 \end{bmatrix}$$

49.]

$$\begin{bmatrix} -4 & -2 & -4 \\ 5 & 3 & 0 \\ 1 & -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 & -3 & 3 \\ -1 & -2 & 2 \\ -1 & 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -4\times 5 + -2\times -1 + -4\times -1 & (-4)\times(-3) + (-2) + (-4)\times 4 & (-4)\times 3 + (-2)\times 2 + (\ \times 1 \\ 5\times 5 + 3\times(-1) + 0\times(-1) & 5\times(-3) + 3\times(-2) + 0\times 4 & + 5\times 3 + 3\times 2 + 0\times 1 \\ 1\times 5 + (-1)\times(-1) + 5\times(-1) & 1\times(-3) + (-1)\times(-2) + 5\times 4 & 1\times 3 + (-1)\times 2 + 5\times 1 \end{bmatrix}$$

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$$\begin{bmatrix} -14 & 0 & -20 \\ 22 & -21 & 21 \\ 1 & 19 & 6 \end{bmatrix}$$

50.] $\begin{bmatrix} -2 & 2 & 3 & 0 & -3 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \\ 5 \\ 5 \end{bmatrix}$

$$\begin{bmatrix} (-2)\times 3 + 2\times 4 + 3\times 1 + 0\times 5 + (-3)\times 5 \end{bmatrix}$$

$$\begin{bmatrix} -10 \end{bmatrix}$$

51.] $\begin{bmatrix} 0 \\ 4 \\ 5 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & -3 & -4 & 0 & -1 \end{bmatrix}$

$$\begin{bmatrix} 0\times 3 & 0\times(-3) & 0\times(-4) & 0\times 0 & 0\times(-1) \\ 4\times 3 & 4\times(-3) & 4\times(-4) & 4\times 0 & 4\times(-1) \\ 5\times 3 & 5\times(-3) & 5\times(-4) & 5\times 0 & 5\times(-1) \\ 3\times 3 & 3\times(-3) & 3\times(-4) & 3\times 0 & 3\times(-1) \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 6 & 0 \\ 12 & -12 & -16 & 0 & -4 \\ 15 & -15 & -20 & 0 & -5 \\ 9 & -9 & -12 & 0 & -3 \end{bmatrix}$$

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52.]
$$\begin{bmatrix} 2 & 5 & -1 & 4 \\ 2 & 2 & -2 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 & -1 & -4 \\ 4 & 1 & 0 & -4 \\ -2 & -2 & -2 & -2 \\ 2 & 4 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2\times 0 + 5\times 4 + (-1)\times(-2) & 2\times(-1) + 5\times 1 + (-1)\times(-2) + 4\times 4 \\ 2\times 0 + 2\times 4 + (-2)\times(-2) & 2(-1) + 2\times 1 + (-2)\times(-2) + (-1)\times 4 \end{bmatrix}$$

$$2\times(-1) + 5\times 0 + (-1)\times(-2) + 4\times 0 \qquad 2\times(-4) + 5\times(-4) + (-1)\times(-2) + 4\times 0$$
$$2\times(-1) + 2\times 0 + (-2)\times(-2) + (-1)\times 0 \qquad 2\times(-4) + 2\times(-4) + (-2)\times(-2) + (-1)\times 0$$

$$\begin{bmatrix} 30 & 21 & 0 & -10 \\ 10 & 0 & 2 & -16 \end{bmatrix}$$

Shake
23/10/24