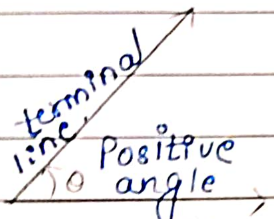
# ★ TRIGONOMETRY

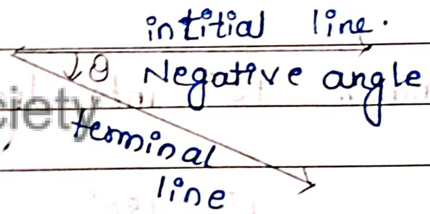
* Syllabus

- Introduction
- Trigonometric ratios
- Transformation Identities
- Inverse trigonometric functions (only elementary topics)

• Angle System :




[fig.1]




[fig.2]



The rotational of initial line as shown in fig. 2 is known as angle if it is in anticlockwise direction then if it is +ve angle otherwise it is -ve angle.

If the right angle can be divided in 90 parts then we have

right angle = $\underline{\underline{90^\circ}}$.

$1^\circ = 60'$

$1' = 60''$

which is known as degree system similarly right angle is divided into 100 grades then we have

$$\text{right angle} = 100^g$$
$$1^G = 100'$$
$$1' = 100''$$

This system is known as grade system like wise

$$\pi r = 180^\circ$$

Or $\pi^c = 180^\circ$

$1^c = \frac{180^\circ}{\pi}$ also $1^\circ = \frac{\pi}{180^\circ}$

• Examples :-

1.) Convert $15^c$ into degree



$$1^c = \frac{180^\circ}{\pi}.$$

$$15^c = \frac{180^\circ}{\pi} \times 15^c$$

$$= \frac{2700^\circ}{\pi}$$

2.) Convert $17^\circ$ into degree.

We know that

$$1^\circ = \frac{\pi}{180^\circ} \times 17$$

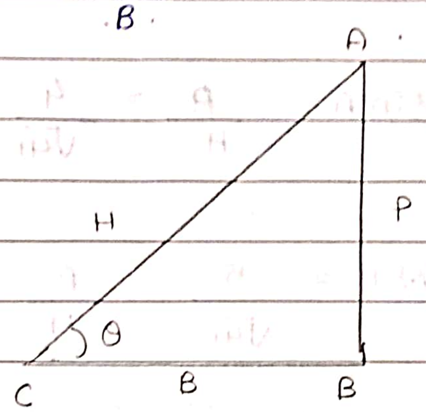
$$17 = \frac{\pi}{180} \times 17 = \frac{17\pi}{180^\circ}.$$

PAGE No. DATE / /

★ Trignometric Ratios :-

There are three Trignometric Ratios given as

$\sin\theta = \frac{P}{h}$ , $\cos\theta = \frac{B}{h}$ , $\tan\theta = \frac{P}{B}$.




The reciprocal Trignometric Ratios are given as

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$\frac{1}{\sin\theta} = \operatorname{cosec}\theta = \frac{h}{P}$

$\frac{1}{\cos\theta} = \sec\theta = \frac{h}{b}$

$\frac{1}{\tan\theta} = \cot\theta = \frac{b}{P}$.

~~Inverse Trignometric Ratios are given by $\sin^{-1}\theta$, $\cos^{-1}\theta$, $\tan^{-1}\theta$, $\cot^{-1}\theta$, $\sec^{-1}\theta$, $\operatorname{cosec}^{-1}\theta$~~

Q] If $\tan A = \frac{4}{5}$ then find other five trignometric ratio.

$AC^2 = AB^2 + BC^2$.

$\tan A = \frac{P}{B} = \frac{4}{5}$.

$\sin A = \frac{A}{H} = \frac{4}{\sqrt{41}}$

$\cos A = \frac{5}{\sqrt{41}} = \frac{B}{H}$

$\operatorname{cosec} A = \frac{\sqrt{41}}{4} = \frac{H}{P}$

$\sec A = \frac{\sqrt{41}}{5} = \frac{A}{B}$



$\cot A = \frac{5}{4} = \frac{B}{P}$.

- Some standard angles :

| functions | $0°$ | $30°$ | $45°$ | $60°$ | $90°$ |
|---|---|---|---|---|---|
| $\sin\theta$ | 0 | $1/2$ | $1/\sqrt{2}$ | $\sqrt{3}/2$ | 1 |
| $\cos\theta$ | 1 | $\sqrt{3}/2$ | $1/\sqrt{2}$ | $1/2$ | 0 |
| $\tan\theta$ | 0 | $1/\sqrt{3}$ | 1 | $\sqrt{3}$ | $\infty$ |
| sec | 1 | $2/\sqrt{3}$ | $\sqrt{2}$ | 2 | $\infty$ |
| cosec | $\infty$ | 2 | $\sqrt{2}$ | $2/\sqrt{3}$ | 1 |

- Identities

1) $\sin^2\theta + \cos^2\theta = 1$

2) $\sec^2\theta - \tan^2\theta = 1$

3) $\csc^2\theta - \cot^2\theta = 1$

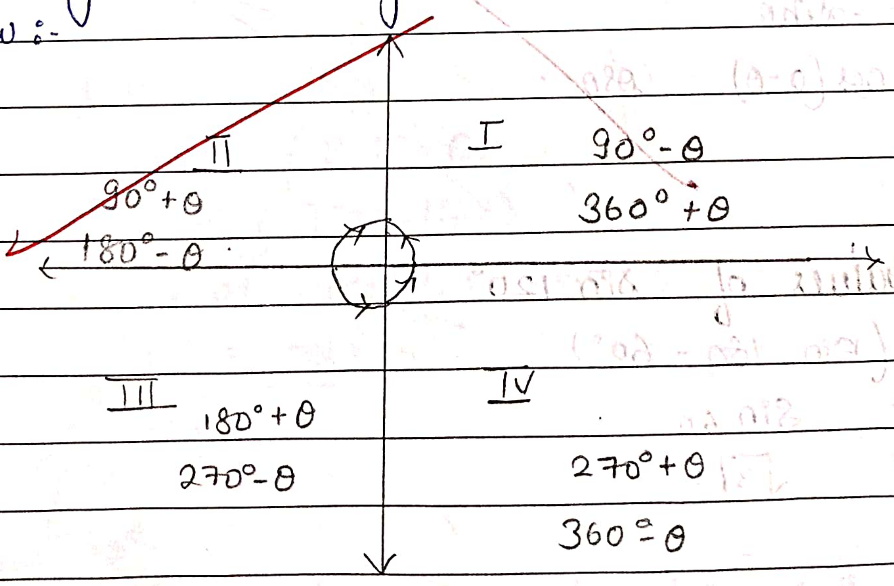
- Allide angles and Co-ratios :-

Allide angles are those angle when the sum or difference of two angles is either is zero or a multiple of $90^\circ$.

Ex :- $30^\circ$, $60^\circ$ are allied angles when the sum or difference of two angles because there sum is $90^\circ$.

The allide angles for $\theta$ are $-\theta$, $90 \pm \theta$, $360^\circ + \theta$.

3] The Angle $\theta$ is major in anticlockwise is as shown below :-



A = All Trigonometric ratios are positive

S = Sine & cosine are positive, rest all are (-ve)

T = Tanθ & cotan are positive, rest all are (-ve).

C = Cosine Sec are positive, rest all are (-ve).

* NOTE :-

1.) At $180° \pm \theta$ and $360° \pm \theta$ trignometric ratio remain same.

2.) At $270° \pm \theta$ and $90° \pm \theta$ trignometric ratio change into there co-ratios.

3.) The co-ratios sinθ is cosθ.
The co-ratios of tanθ is cotθ.
The co-ratios of cosecθ is secθ & vice versa.



Eg:-

1.) $\sin(90° + \theta) = \cos\theta$

2.) $\sin(270° - \theta) = -\cos\theta$

3.) $\cos(180° - \theta) = -\cos\theta$

4.) $\sin(-\theta) = -\sin\theta$

5.) $\cos(-\theta) = \cos(0 - \theta) = \cos\theta$.

1.) Find the values of $\sin 120°$

→ $\sin 120° = (\sin 180 - 60°)$

$= \sin 60$

$= \sqrt{3}/2$

DATE / /

2.] $\cos\theta = 210°$

$\cos(270° - 210°)$

$\cos 60°$

$= \frac{1}{2}$

* H.W

1.) $\cos 120°$

$\cos 120°$

$= \cos(180° - 60°)$

$= \cos(-60°)$

$= -\left(\frac{1}{2}\right)$



2.) $\tan 210°$

$\tan(180° + 30°)$

$\tan(210°)$

$\tan 30° = 1/\sqrt{3}$

3.) $\cot 150°$

$\cot(\pi - \theta)$

$\cot(180° - 150°)$

$\cot(-30°)$

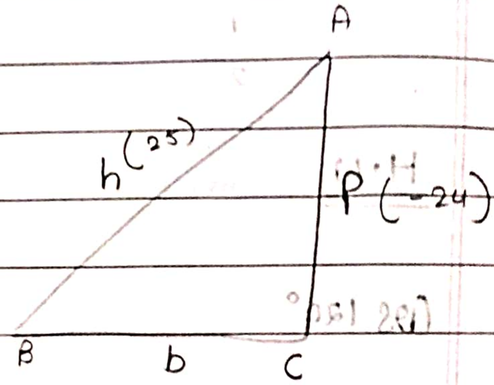
$= \underline{-\sqrt{3}}$

4.) $\sec 300°$

$\sec(360° - 300) = \boxed{\sec 60° = 2}$

Q.] If $\sin\theta = \frac{-24}{25}$ and $\theta$ lies in $4^{th}$ quardant then find $\cos\theta$ and $\tan\theta$.

→ $\sin\theta = \frac{-24}{25}$ lies in II quadrant.




$\sin\theta = \frac{P}{h}$.

according to pythagoras

$AC^2 = AB^2 + BC^2$.

$(25)^2 = (24)^2 + BC^2$

$625 = 576 + BC^2$

$625 - 576 = BC^2$

$= 49 = BC^2$

$\sqrt{49} = BC$

$7 = BC$.



1.→ $\cos\theta = \frac{B}{h}$.

$\boxed{\cos\theta = \frac{+7}{25}}$.

2.→ $\tan\theta$.

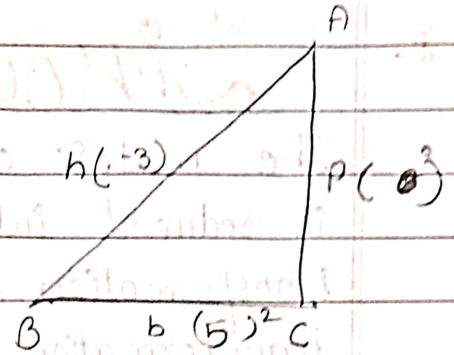
$\tan\theta = \frac{P}{B}$

$\boxed{\tan\theta = \frac{-24}{7}}$

Q.] If $\theta$ is located in third quardant and $\cos\theta = \frac{-3}{5}$ then find $\sin\theta$, $\tan\theta$ and $\cot\theta$.

→ $\cos\theta = \frac{-3}{5}$

$\cos\theta = \frac{B}{h}$




according to pythagoras.

$\cancel{BC^2 = (AB^2) + (AC^2)}$
$\cancel{BC^2 = -3^2 + 5^2}$
$\cancel{BC^2 = 9 + 25}$
$\cancel{BC^2 = 34}$
$\cancel{BC = \sqrt{34}}$

$(5)^2 = (-3)^2 + (x^2)$
$(25) = 9 + x^2$
$16 = x^2$
$x = 4$

$P = 4$
$H = 5$
$B = -3$

1.) $\sin\theta = \frac{P}{h}$

$\boxed{\frac{\cancel{34}}{-\cancel{3}}}$ $\frac{4}{5}$

2.) $\tan\theta = \frac{P}{B}$

= $\boxed{\frac{\cancel{3}4}{\sqrt{\cancel{34}}\,3}}$ $\frac{\cancel{4}}{\cancel{3}}$ $\frac{-4}{-3}$

3.) $\cot\theta$ $\frac{b}{P}$

$\boxed{\frac{\cancel{\sqrt{34}}\,-3}{\cancel{3}\,4}}$

NotesSociety

DATE / /

UNIT – I

# *. Linear Equation

- The matrix of the coffecients of x, y, z is reduced into echelon from by elementary row transformation ~~and~~ at the end of the row transformation the value of z is calculated from the last equation and value of y and value of x are calculated by the backward substitution.

ex:

$$x - y + 2z = 3$$
$$x + 2y + 3z = 5$$
$$3x - 4y - 5z = -13$$

## *. Types of Linear equation

1) consistent :- A system of equation is said to consistent if they have 1 or more solution that is.

$$x + 2y = 4 \qquad\qquad x + 2y = 4$$
$$3x + 2y = 2 \qquad\qquad 3x + 6y = 12$$

unique $sol^n$      Infinite $sol^n$.

2) Inconsistent equation :- If a system of equation has no solution it is said to be inconsistent.

$$x + 2y = 4$$
$$3x + 6y = 5$$

1/10/24

# Trignometry

4) Compound angles :- The algebric sum or diffnt between the angles is known as compound angles. These angles are used to solve the algebric sum or difference of trignometric function which is stated as

(2)(i) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

(ii) $\cos(A-B) = \cos A \cos B - \sin A \sin B$.

(3)(i) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

1.) $\sin(A+B) = \sin A \cos B + \cos A \cdot \sin B$.

(ii) $\sin(A-B) = \sin A \cos B - \cos A \sin B$.

(ii) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

ex:- Find sin 15° and sin 75°.



$\sin(45° - 30°)$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\sin(45° - 30°) = \sin 45 \cos 30° - \cos 45° \sin 30°$

$= \left(\frac{1}{\sqrt{2}}\right) \times \left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \boxed{\frac{\sqrt{3}}{2\sqrt{2}}}$

$\sin 75^\circ$

$\sin(45^\circ+30^\circ)$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(45+30^\circ) = \sin 45\cos 30 + \cos 45\sin 30$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

Q] Find the value of tan 15° and sin 105°.



$\tan 15^\circ$

$\tan(45-30)$

$\tan(A-B) = \frac{\tan A - \tan B}{1-\tan A\tan B}$

$\tan(45^\circ-30^\circ) = \frac{\tan 45^\circ - \tan 30^\circ}{1+\tan 45\tan 30}$

$= \frac{1 - 1/\sqrt{3}}{1+1(1/\sqrt{3})}$

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PAGE No.
DATE / /

$\sin 105°$

$\sin(60+45°)$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(60+45°) = \sin 60 \cos 45° + \cos 60° \sin 45°$

$= (\sqrt{3}/2)(1/\sqrt{2}) + (1/2)(1/\sqrt{2})$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

Q] $\cos 105°$ $\tan 105°$

$\cos(105°)$

$\cos(60+45°)$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(60+45°) = \cos 60 \cos 45 - \sin 60 \sin 45$

$= (1/2)(1/\sqrt{2}) - (\sqrt{3}/2)(1/\sqrt{2})$

$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$.

$\tan 105°$

$\tan(105°)$

$\tan(60+45°)$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(A+B) = \frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45} = \frac{\sqrt{3}+1}{1-\sqrt{3}(1)}$

$= \frac{\sqrt{3}+1}{1-\sqrt{3}}$

11/10/24

5.1 Inverse trigonometric functions :

If $A \to B$ is a one & onto function then $f^{-1} : B \to A$ is the inverse function of $f : A \to B$

We can note that if $y = f(x)$

$\Rightarrow f^{-1}(y) = x$.

There are six inverse trignometric ratios whose domain & principal value branch is given as :-

(i) $y = \sin^{-1}x$

Domain $[-1, 1]$

principal value branch = ~~$[0, \pi]$~~ $[-\pi/2, \pi/2]$

(ii) $y = \tan^{-1}x$

Domain = $(-\infty, \infty)$

principal value branch = $[-\pi/2, \pi/2]$



(B) Some imp graphical views of trigonometric ratios & inverse trigonometric ratios :

(i) Graph of $y = \sin x$

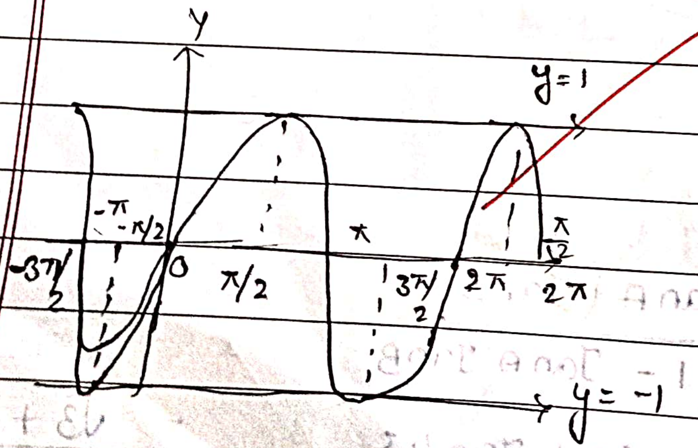
Domain = $(-\infty, \infty)$

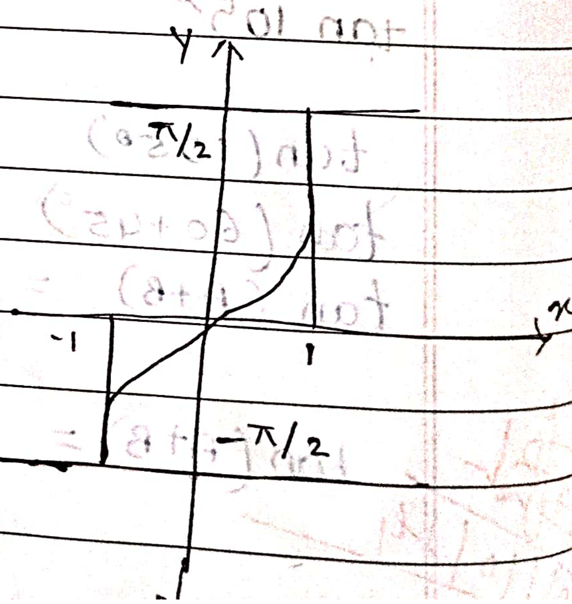
Range = $[-1, 1]$

Graph of $y = \sin^{-1}x$

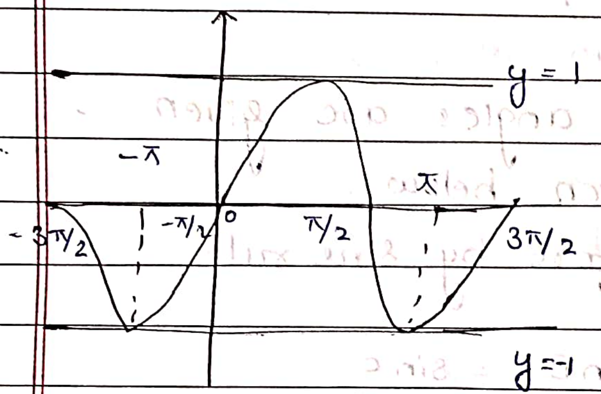
Domain = $[-1, 1]$

Range = $[-\pi/2, \pi/2]$

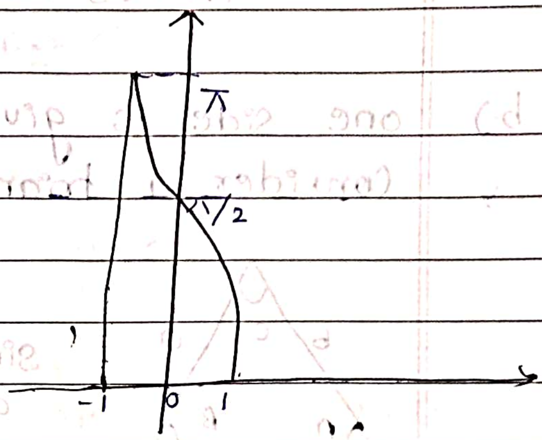


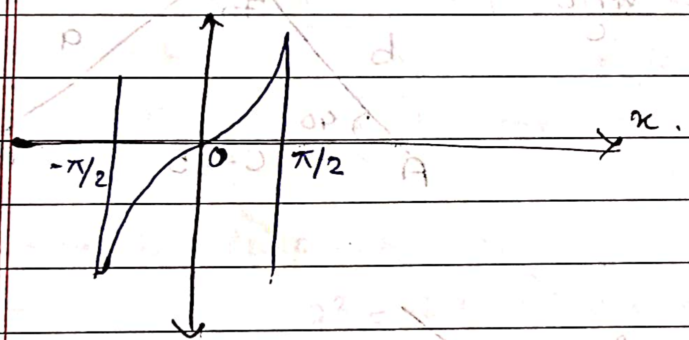
(iii) Graph of $y = \cos x$
Domain $= (-\infty, \infty)$
Range $= [-1, 1]$



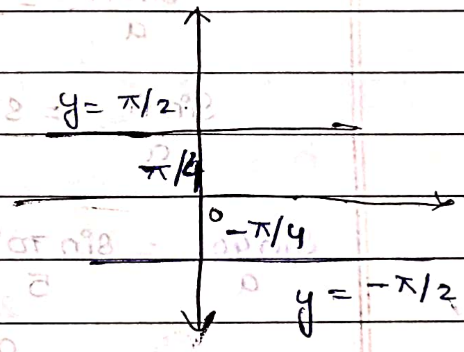

Graph of $y = \cos^{-1} x$
Domain $= [-1, 1]$
Range $= [0, \pi]$




Graph of $y = \tan x$
Domain $= R - \{x : x = (2n+1)\pi/2, n \in z\}$
Range $= (-\infty, \infty)$




Graph of $y = \tan^{-1} x$
Domain $= R \ (-\infty, \infty)$
Range $= (-\pi/2, \pi/2)$






☆ Solution of a triangle :

~~If the three angles in a triangle & three sides~~
If three elements are given then the process of finding the other elements is known as the 'solution of a Triangle'.

a.) No side is given / two angles are given

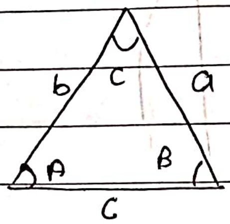
→ we know that the sum of three interior angles of a triangle is equal to 180°; i.e

$$\angle A + \angle B + \angle C = 180°.$$

b.) one side is given / two angles are given

→ Consider a triangle given below :

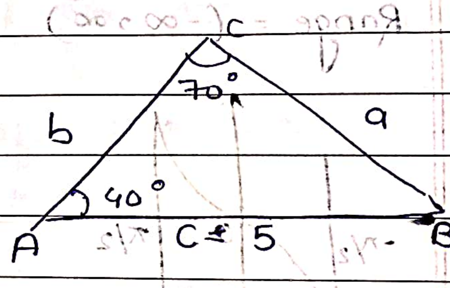
Therefore by sine rule




$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

eg:- find the length of a, where $\angle A = 40°$ $\angle C = 70°$ $c = 5$






$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

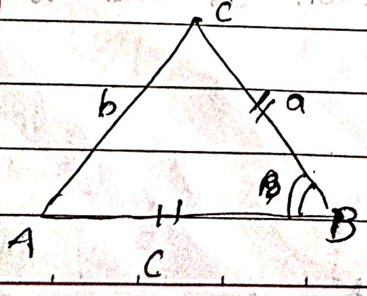
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 40°}{a} = \frac{\sin 70°}{5}$$

$$a = \frac{\sin 40°}{\sin 70°} \times 5 \qquad a = 3.42.$$

c.) Two sides & one included angle is given

Therefore by cosine rule




$$a = \sqrt{b^2 + c^2 - 2bc \cos A}$$

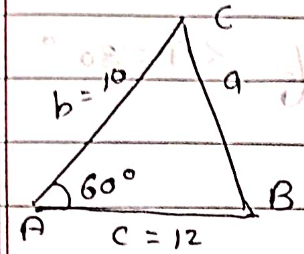
$$b = \sqrt{a^2 + c^2 - 2ac \cos B}$$

$$c = \sqrt{a^2 + b^2 - 2ab \cos C}$$

PAGE No.

DATE / /

eg:-




using cosine rule

$$a = \sqrt{b^2 + c^2 - 2bc \cos A}$$

$$a = \sqrt{(10)^2 + (12)^2 - 2(10)(12) \cos 60^\circ}$$

$$a = \sqrt{100 + 144 - 240 \cos 60^\circ}$$

$$a = \sqrt{4 \cos 60^\circ}$$

$$a = \sqrt{4 \times 1/2}$$

$$\boxed{a = \sqrt{2}}$$

(d) Three sides are given & no angle is given:
In this case, both sine & cosine rule can be used. i.e $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ [the sine Rule]



&

$$a = \sqrt{b^2 + c^2 - 2bc \cos A}$$

$$b = \sqrt{c^2 + a^2 - 2ac \cos B}$$

$$c = \sqrt{a^2 + b^2 - 2ab \cos C}$$

[the cosine Rule]

which implies

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$2bc \cos A = b^2 + c^2 - a^2$$

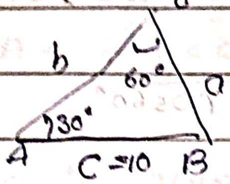
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Similarly,

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

find the ~~angle~~ length of side A if $\angle A = 30°$ & $\angle C = 60°$ & $C = 10$.




$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 30°}{a} = \frac{\sin 60}{10}$$

$$a = \frac{\sin 30 \times 10}{\sin 60}$$

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$$a = \frac{1/2}{\sqrt{3}/2} \times 10$$

$$a = \frac{1}{2} \times \frac{2}{\sqrt{3}} \times 10$$

$$= \frac{2 \times 10}{2\sqrt{3}}$$

$$a = \frac{10}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$a = \frac{10\sqrt{3}}{\cancel{3}}$$

Q] Find the length of side b < A = 60° and < B = 90° C = 5

→

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 60°}{a} = \frac{\sin 90°}{c}$$

~~Sin~~

$$b = \frac{\sin 60}{\sin 90} \times 5$$

$$b = \frac{\sin \sqrt{3}/2}{1} \times 5$$

$$b = \frac{\sin \sqrt{3}/2 \times 5}{\frac{\cancel{7}}{1}}$$

$$b \quad \frac{10\sqrt{3}}{3}.$$

Q]

* 2 sides & 1 < is given ∴ by cosine rule

1.) $a = \sqrt{b^2 + c^2 - 2bc \quad \cos A}$

$b = \sqrt{a^2 + c^2 - 2ac \quad \cos B}$

$c = \sqrt{a^2 + b^2 - 2ab \quad \cos C}$

Q. if B cos = 10, C = 12 & < A = 60° find l of A.

$a = \sqrt{b^2+c^2-2bc\cos A}$

$\sqrt{10^2+12^2-2\times10\times12\cos A}$

$= \sqrt{100+144-240\cos A}$

$= \sqrt{244-240\cos A}$

$= \sqrt{4\times\cos 60^\circ}$

$= 4\times 1/2$

$= \sqrt{2}$



* If 3 side are given & No angle is given In This case both Sin & Cosin rule can be used

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ = Sin rule

Cosin rule ③

$a = \sqrt{b^2+c^2-2bc\cos A}$

$b = \sqrt{a^2+c^2-2ac\cos B}$

$c = \sqrt{a^2+b^2-2ab\cos C}$

PAGE No.
DATE / /

formulas

$a^2 = b^2+c^2 - 2bc \cos A$

$2bc \cos A = b^2+c^2 - a^2$

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

*. If $a=10$, $b=8$ and $c=7$ then find angle C.

$$\angle C = \frac{a^2+b^2-c^2}{2ab}$$

$$\angle C = \frac{10^2+8^2-7^2}{2(10)(8)} = \frac{100+64-49}{20\times 8}$$

$$= \frac{164-49}{160}$$

$$= \frac{\cancel{115}}{\cancel{160}} = \frac{23}{32}$$

$= \underline{1.39}$

DATE / /

1 Find the length of A $\angle a = 60^\circ$ $\angle b = 30^\circ$ B = 6
2 find the ~~and~~ length of A $\angle a = 70^\circ$ $\angle b = 90^\circ$ C = 9
3 find the length of Ba $\angle a = 45^\circ$ $\angle b = 60^\circ$ A = 7
4 find the length of C $\angle a = 60^\circ$ $\angle C = 45^\circ$ A = 5
5 find the length of C $\angle b = 60^\circ$ $\angle C = 90^\circ$ B = 8
6 find the length of B $\angle b = 65^\circ$ $\angle C = 80^\circ$ C = 3

Q1) —— 9

→

A = ?
$\angle a = 60^\circ$
$\angle b = 30^\circ$
B = 6

using

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 60^\circ}{a} = \frac{\sin 30^\circ}{6} \quad = \frac{\sqrt{3}/2}{a}$$

$$a = \frac{\sin 60^\circ \times 6}{\sin 30^\circ}$$

$$a = \frac{\sqrt{3}/2}{1/\cancel{2}\,1} \times \cancel{6}\,3$$

$$a = \frac{\sqrt{3}/6}{1}$$

NotesSociety

DATE / /

Q2.] $A = ?$

$\angle a = 70^\circ$

$\angle b = 60^\circ$

$C = 7$

using

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 70^\circ}{a} = \frac{\sin 60^\circ}{b} = 7$$

$$A^\circ = \frac{\sin 70^\circ}{\sin 60^\circ} \times 7$$

$A =$ ~~0.96~~ $7.595$.

3) $$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 45^\circ}{7} = \frac{\sin 60^\circ}{b} \Rightarrow b = \frac{\sin 60^\circ \times 7}{\sin 45^\circ} = \frac{\frac{\sqrt{3} \times 7}{2}}{\frac{1}{\sqrt{2}}}$$

$$= \frac{\sqrt{3}}{2} \times 7 \times \frac{\sqrt{2}}{1}$$

$$= \frac{\sqrt{6} \times 7}{2} = \frac{7\sqrt{6}}{2}$$

Q.] $\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin 60^\circ}{5} = \frac{\sin 45^\circ}{c}$

$c = \frac{\sin 45^\circ \times 5}{\sin 60^\circ} = \frac{\frac{1}{\sqrt{2}} \times 5}{\sqrt{3}/2} = \frac{5}{\sqrt{2}} \times \frac{2}{\sqrt{3}} = \frac{10}{\sqrt{6}}$

Q.] $\frac{\sin B}{b} = \frac{\sin C}{c} = \frac{\sin 60^\circ}{8} = \frac{\sin 90^\circ}{c}$

$c = \frac{\sin 90^\circ \times 8}{\sin 60^\circ} = \frac{1 \times 8}{\sqrt{3}/2} = \frac{8 \times 2}{\sqrt{3}} = \frac{16}{\sqrt{3}}$.

Q.] $\frac{\sin B}{b} = \frac{\sin C}{c} = \frac{\sin 65^\circ}{b} = \frac{\sin 80^\circ}{3}$

$b = \frac{\sin 65^\circ \times 3}{\sin 80^\circ} = \frac{0.9063 \times 3}{0.98481} = \frac{2.7189}{0.98481}$



Q.] No side given 2 angles given

1.) $\angle A = 45^\circ$ $\angle B = 45^\circ$ $\angle C = ?$

$\angle A + \angle B + \angle C = 180^\circ$

$45^\circ + 45^\circ + \angle C = 180^\circ$

$\angle C = 180^\circ - 90^\circ$

$\angle C = 90^\circ$

2.) $\angle B = 30^\circ$ $\angle C = 90^\circ$ $\angle A ?$

$\angle A + \angle B + \angle C = 180^\circ$

$\angle A + 30^\circ + 90^\circ = 180^\circ$

$\angle A = 180^\circ - 120^\circ$

$\angle A = 60^\circ$,

3.) $\angle A = 90^\circ$ $\angle C = 60^\circ$ $\angle B = ?$

$\angle A + \angle B + \angle C = 180^\circ$

$90^\circ + \angle B + 60^\circ = 180^\circ$

$\angle B = 180^\circ - 150^\circ$

$\angle B = 30^\circ$.

4.) $\angle C = 45^\circ$ $\angle A = 90^\circ$ $\angle B = ?$

$\angle A + \angle B + \angle C = 180^\circ$

$90^\circ + \angle B + 45^\circ = 180^\circ$

$\angle B = 180^\circ - 135^\circ$

$\angle B = 45^\circ$.

Q5.] $\angle A = 95^\circ$ $\angle B = 25^\circ$, $\angle C = ?$

$\angle A + \angle B + \angle C = 180^\circ$

$95^\circ + 25^\circ + \angle C = 180^\circ$

$\angle C = 180^\circ - 120^\circ$

$\angle C = 60^\circ$.

Q.] $\angle B = 90^\circ$ $\angle C = 65^\circ$ $\angle A = ?$

$\angle A + \angle B + \angle C = 180^\circ$

$\angle A + 90^\circ + 65^\circ = 180^\circ$

$\angle A = 180^\circ - 155^\circ$

$\angle A = 95^\circ$.

*. Two sides one angle

1.) $b = 4$ $c = 3$ $\angle B = 60^\circ$ $a = ?$

$a = \sqrt{b^2 + c^2 - 2bc \cos B}$

$a = \sqrt{4^2 + 3^2 - 2(4)(3) \cos 60^\circ}$

$a = \sqrt{16 + 9 - 24 \cos 60^\circ}$

$a = \sqrt{1 \cos 60^\circ}$

$a = \sqrt{1/2}$

4.) $a = 6$ $b = 7$ $\angle C = 90$ $c ?$

$c = \sqrt{a^2 + b^2 - 2ab \cos C}$

$c = \sqrt{6^2 + 7^2 - 2(6)(7) \cos 90^\circ}$

$c = \sqrt{36 + 49 - 84 \cos 90^\circ}$

$c = \sqrt{\cdot 1 \cos 90^\circ}$

$c = \sqrt{0} = 0$

2.) $c = 7$ $b = 9$ $\angle A = 45^\circ$ $a ?$

$a = \sqrt{b^2 + c^2 - 2bc \cos A}$

$a = \sqrt{9^2 + 7^2 - 2(9)(7) \cos A}$

$a = \sqrt{81 + 49 - 126 \cos 45^\circ}$

$a = \sqrt{4 \cos 45^\circ}$

$a = \sqrt{\frac{4 \times 1}{\sqrt{2}}}$ $a = \frac{\sqrt{4}}{\sqrt{2}}$ $a = \sqrt{2}$

5.) $a = 5$ $b = 6$ $\angle C = 30$ $c = ?$

$c = \sqrt{a^2 + b^2 - 2ab \cos C}$

$c = \sqrt{5^2 + 6^2 - 2(5)(6) \cos 30^\circ}$

$c = \sqrt{25 + 36 - 60 \cos 30^\circ}$

$c = \sqrt{1 \cos 30^\circ}$

$c = \sqrt{1 \times \frac{\sqrt{3}}{2}}$ $\boxed{c = 3/2.}$

3.) $a = 8$ $c = 10$ $\angle B = 60^\circ$ $b = ?$

$b = \sqrt{c^2 + a^2 - 2ac \cos B}$

$b = \sqrt{10^2 + 8^2 - 2(8)(10) \cos 60^\circ}$

$b = \sqrt{100 + 64 - 160 \cos 60^\circ}$

$b = \sqrt{4 \cos 60^\circ}$

$b = \sqrt{\frac{4 \times 1}{2}} = b \frac{\sqrt{4}\,2}{\cancel{2}\,1}$

$\boxed{b = \sqrt{2}}$

6.) $a = 9$ $c = 9$ $\angle B = 45^\circ$ $b = ?$

$b = \sqrt{c^2 + a^2 - 2ac \cos B}$

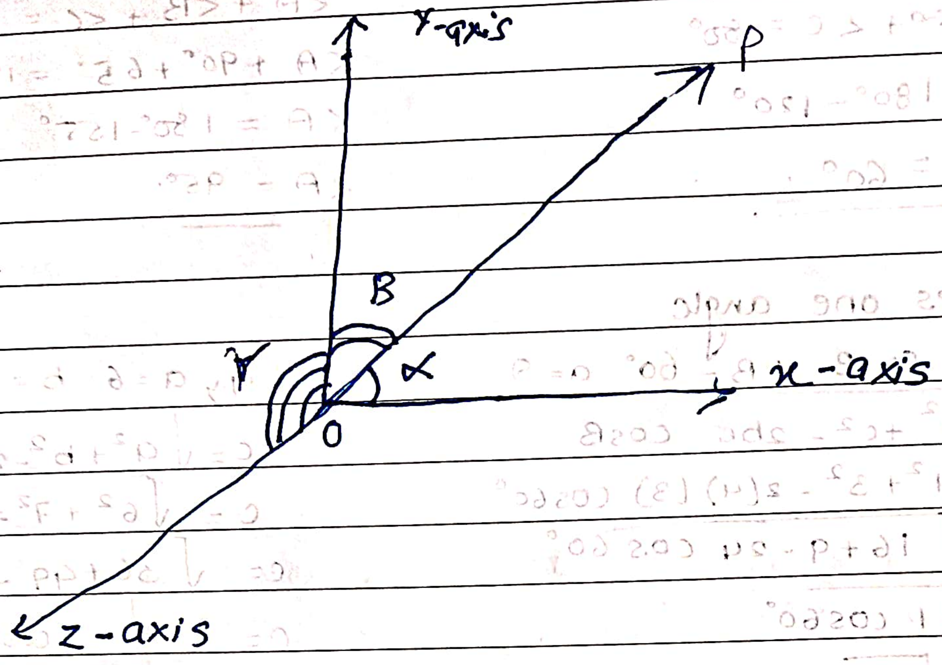
$b = \sqrt{9^2 + 9^2 - 2(9)(9) \cos B}$

$b = \sqrt{81 + 81 - 162 \cos 45^\circ}$

$b = \sqrt{\cos 45^\circ}$

$\boxed{= \frac{1}{\sqrt{2}}}$

## * Direction cosine Ratio *






Suppose O & P be a vector such that it makes angle $\alpha$ with x-axis; angle $\beta$ with y axis and angle $\gamma$ with z-axis; $\cos\alpha$, $\cos\beta$ & $\cos\gamma$ are known as direction cosine. These are also denoted by ~~single~~ 'l', 'm', 'n' respectively. and having relation $l^2+m^2+n^2=1$. If 'A', 'B', 'C' are direction ratio then we have

$$l^2+m^2+n^2-1$$

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$$

$$l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}} \qquad m = \pm \frac{b}{\sqrt{a^2+b^2+c^2}}$$

PAGE No.
DATE / /

3.) $n = \pm \frac{c}{\sqrt{1^2+b^2+c^2}}$

NOTE :- The no proportional to a direction using cosine of a vector are called direction ratio.

Ex:- find the direction cosine of a vector making angle 30 & 60, 90 with x axis, y, z axis.

$\alpha = 60$
$B = 30°$
$\gamma = 90°$.

NotesSociety

Direction Cosine ratio $\cos\alpha = \cos 60° = \frac{1}{2}$.

" " " $\cos B = \cos 30° = \frac{\sqrt{3}}{2}$

" " " $\cos\gamma = \cos 90° = 0$

find the direction cosine of a line having its direction ratio as 2, 1 & -2

$a = 2$ $\quad l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}}$

$b = 1$

$c = -2$.

$l = \pm \frac{2}{\sqrt{4+1+4}}$

$l = \pm \frac{2}{\sqrt{9}}$ $\quad l = \pm \frac{2}{3}$

2) $m = \pm \dfrac{b}{\sqrt{9}}$

$m = \pm \dfrac{1}{3}$

3) $n = \pm \boxed{\dfrac{-2}{3}}$

NOTE :- The direction ratio of a line joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$

direction cosine are $l = \dfrac{x_2 - x_1}{|\vec{PQ}|} = \dfrac{x_2 - x_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$

$m = \dfrac{y_2 - y_1}{|\vec{PQ}|} = \dfrac{y_2 - y_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$

$n = \dfrac{z_2 - z_1}{|\vec{PQ}|} = \dfrac{z_2 - z_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$

NotesSociety

1.] $a = 4 \quad b = 3 \quad c = 5$

$l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}}$ $\quad l = \frac{4}{\sqrt{16+9+25}}$

$l = \frac{4}{\sqrt{50}} = \frac{4}{5\sqrt{2}}$

$m = \pm \frac{3}{5\sqrt{2}}$ $\quad n = \pm \frac{5}{5\sqrt{2}}$

2.] $a = 6, \quad b = 4, \quad c = 6$



$l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}}$

$l = \pm \frac{6}{\sqrt{36+16+36}}$ $\quad l = \pm \frac{6}{\sqrt{98}}$

$m = \pm \frac{4}{\sqrt{98}}$ $\quad n = \pm \frac{6}{\sqrt{98}}$

3.] $a = -4, \quad b = -3, \quad c = 4$

$l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}} = \pm \frac{-4}{\sqrt{16+9+16}} = \frac{-4}{\sqrt{41}}$

$$m = \frac{-3}{\sqrt{41}} \qquad n = \frac{-4}{\sqrt{41}}$$

4) $a = 9$, $b = 6$, $c = 3$

$$l = \frac{9}{\sqrt{81+36+9}} = \frac{9}{\sqrt{134}}$$

$$m = \frac{6}{\sqrt{134}}$$

$$n = \frac{3}{\sqrt{134}}$$

5) $a = 11$, $b = 11$, $c = 11$

NotesSociety

$$l = \frac{11}{\sqrt{121+121+121}} = \frac{11}{\sqrt{363}}$$

$$m = \frac{11}{\sqrt{363}}$$

$$n = \frac{11}{\sqrt{363}}$$

15/10/24

Complete

Unit III :-

# DIFFERENTIAL CALCULUS

1) Continuity of a function :-

The function $f(x)$ is said to be continous at point $x=c$ if $\lim f(x) = f(c)$

In other words,

for any $\epsilon > 0$ there exists $\delta > 0$ such that $|f(x) - f(c)| < \epsilon$ whenever $|x-c| < \delta$

The function is said to be continuous in an interval $[a, b]$, if it is continuous at every point of interval.

ex:- Examine the continuity of a function $f(x) = 2x^2 - 1$ at $x = 3$.

$f(3) = 2(3)^2 - 1$

$= 2(9) - 1$



$= 18 - 1$

$f(3) = 17$

$\lim_{x \to 3} f(x) = \lim_{x \to 3} (2x^2 - 1) = \lim_{\to} (2 + 9 - 1) =$

$\lim_{x \to 3} (17)$

$[\lim_{x \to c} f(x)]$

i.e.

$\lim_{x \to 3} (2x^2 - 1) = f(3)$.

ex:- Examine the continuity of $f(x) = x^3 - 2x + 1$ at $x = 2$.

DATE

$$f(2) = x^3 - 2x + 1$$
$$= (2)^3 - 2(2) + 1$$
$$= 8 - 4 + 1$$
$$= 4 + 1$$
$$f(2) = 5$$

Now,

$$\lim_{x \to 2} f(x) = \lim (x^3 - 2x + 1) = \lim_{x \to 2} (2^3 - 2(2) + 1)$$
$$= \lim_{x \to 2} (5)$$

i.e.

$$\therefore \lim_{x \to 2} (x^3 - 2x + 1) = f(2).$$