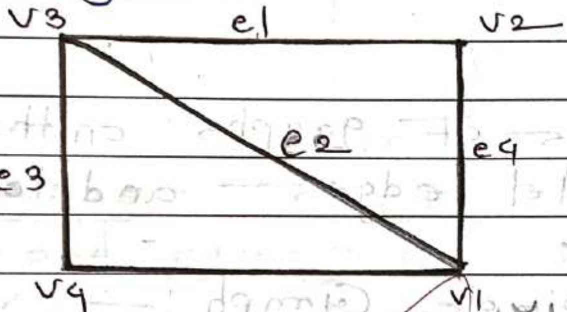
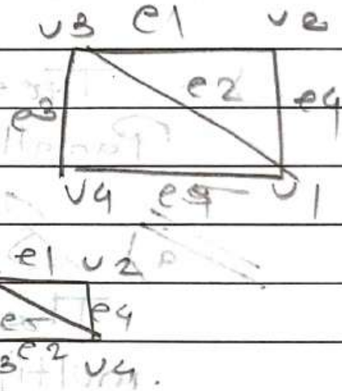
PAGE NO.
DATE:

28/01/2024 2nd sem

Unit - 1 Graph Theory

- Defination:- The pictorial representation of digit which are Connected by links is known as a graph where the objects are known as edges.



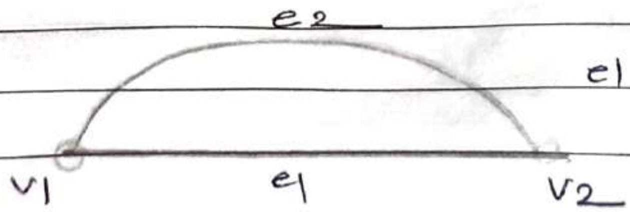


Graph

Vertices = {v1, v2, v3, v4}
Edges = {e1, e2, e3, e4}

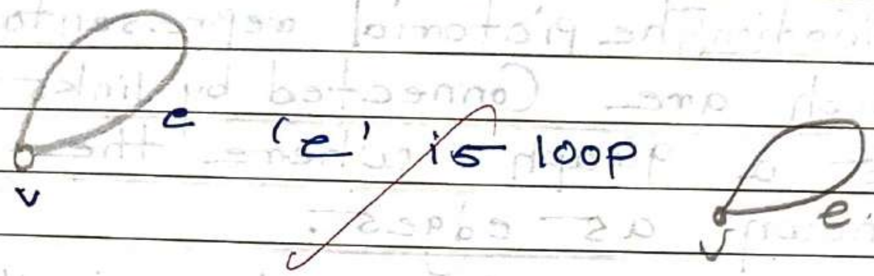
① Parallel edges / Multiple edges :-

If there are more than one edges between any Pair vertices then the edges are known as Parallel edge or multiple edge.



e1 and e2 are Parallel / Multiple edges

ii) loop :- The edge whose starting and end vertex is same is known as a loop.

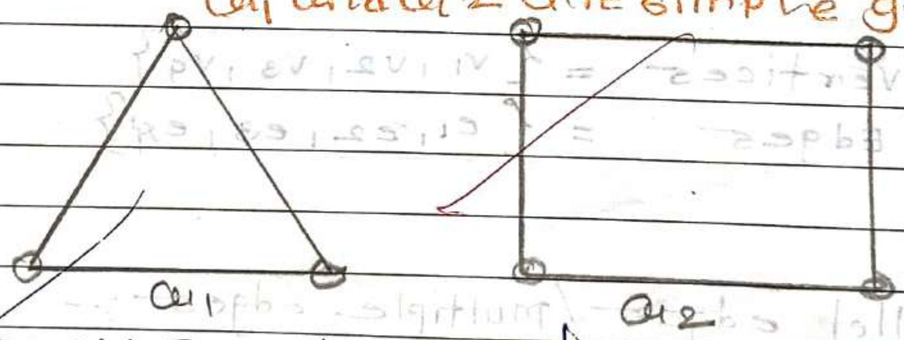


'e' is loop

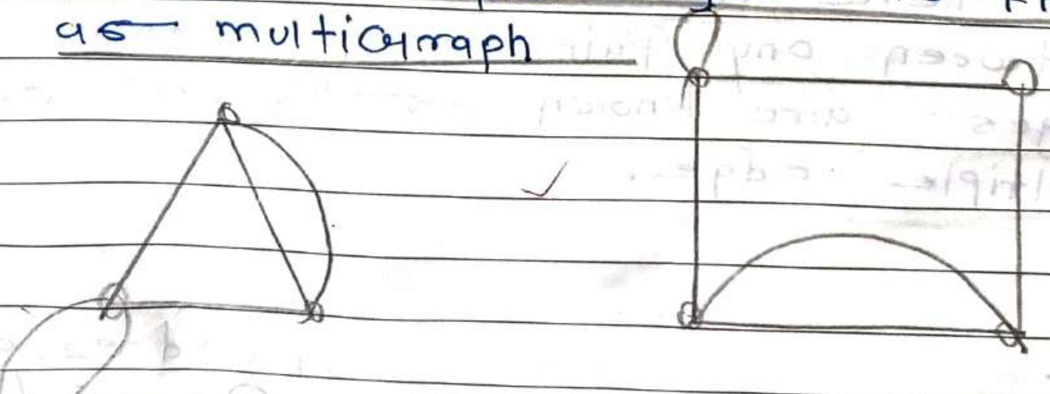
> Tyes of graphs on the basis of Parrallel edges and loops.

a) Simple Graph :- The Graph which does not Contain multiple edges or not.

Gı and G2 are simple graph.



b) Multi Graph :- The Graph which Contain multiple edges is known as multiGraph

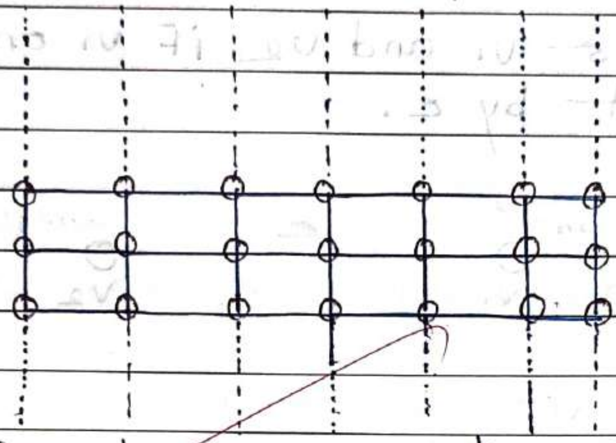


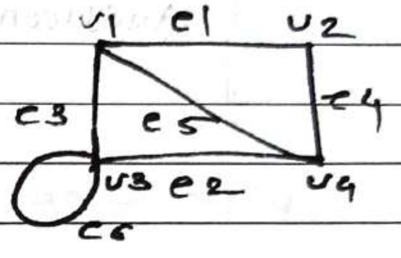
VAIBHAVI

PAGE NO.

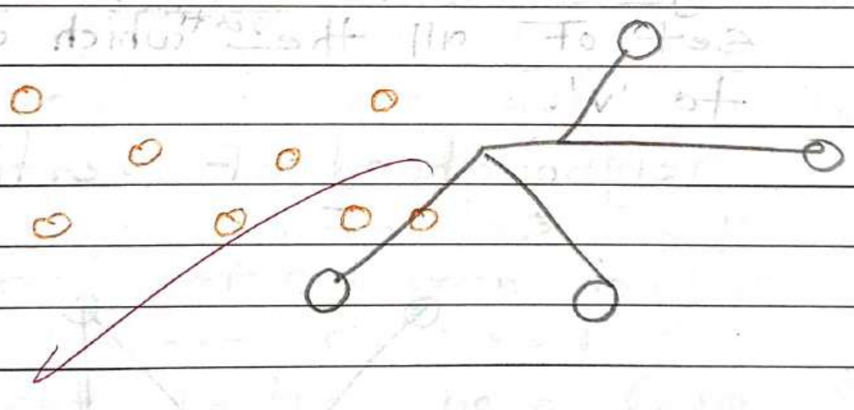
DATE:

c) Psedo Graph :— The Graph which Contain multiple edges as well as loops is known as Pseudo Graph.

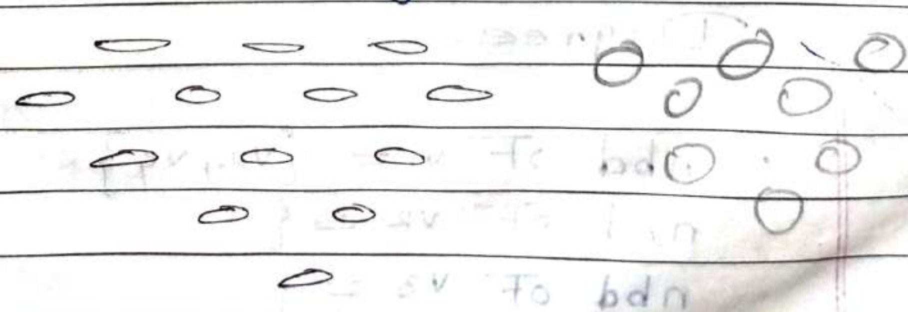




d) Finite Graph :— The Graph is said to be if it has Finite number of verticies and edges otherwise InFinite Graph.



e) Null Graph :— The Graph is said to be Null Graph if it has any number of vertices without edge.



6) Incidence and Neigbourhood :-

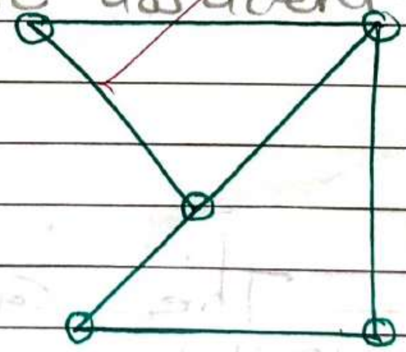
a) Incidence :-

An age 'e' said to be Incidents on Vertices v_1 and v_2 if v_1 and v_2 are Aadjacent by e.

b) Neigbourhood :-

neigboorhood of a vertices 'v' is the set of all the vertices which are adjacent to 'v'.

neigbourhood of vertices 'v' is the set of all the vertices which are adjacent to 'v'.



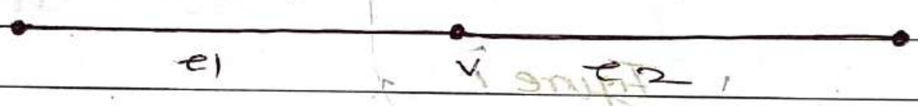
Degree

nbd of v_1 = $\{v_2, v_4\}$
nbd of v_2 = {
nbd of v_3 = {

4) trivial Graph :—
The Graph is said to be trivial Graph if it has a single vertex without edges.

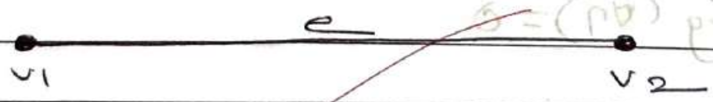
5) Adjacanci
There are two types of adjacancy

a) Adjacent edges :— Two edges are said to be adjacent if they have Common vertex.



e1 and e2 are adjacent edges

b) Adjacent vertices :—
Two vertices are said to be adjacent if they have Common



v1 and v2 are adjacent vertices.

7 February / 2024 Monday

* Degree of a Vertices

a) The Degree of vertiex 'v' is the no of edges Incident on 'v'.

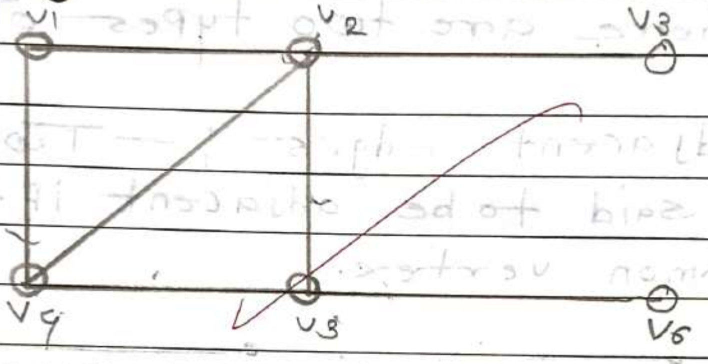


Figure 1

deg $(v_1) = 2$
deg $(v_2) = 4$
deg $(v_3) = 1$
deg $(v_4) = 3$
deg $(v_5) = 3$
deg $(v_6) = 1$
deg $(v_7) = 0$

b) Pendent Vertex

The vertex 'v' is said to Pendent vertex if it is '1'

In the Figure 1 'v_3' and v_6' are Pendent vertices.

PAGE NO.

DATE:

c) Isolated vertex

The vertex 'v' is said to the Isolated vertex if it is Degree is '0'

In the Figure-1 'v' is an Isolated vertex.

g) Degree sequences of a Graph

a) The degree Sequence of a Graph is the Collection of degree of all the vertices of the Graph arrange in either Increasing order or Decreasing Order.

The sequence of a Graph in Figure-1 is given as

0, 1, 1, 2, 3, 3, 4

OR

4, 3, 3, 2, 1, 1, 0

b) Hand-shaking Theorem / Fundamental Theorem of Graph theory.

statement :— it's states that the sum of degrees of all the vertices of a graph is twice of number of edges in Figure-1 we have.

$0+1+1+2+3+3+4 = 14 = 2\times 7$.

where '7' is the number of edges

Hence, Handshaking theorem is verified

From this graph in Figure-1

mathematically we have.

$$\sum_{v \in V} \deg(v) = 2m$$

where 'm' is number of edge

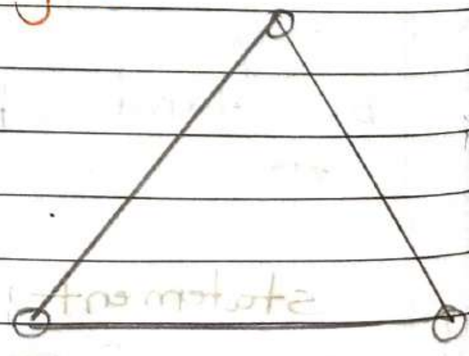
Examples

1) For a simple graph with 'n' vertices the minimum possible of edges is

$$\frac{n(n-1)}{2}.$$

Answer:- For $n=3$

Max Possible no. of edges



$$\text{edges} = \frac{n(n-1)}{2}$$

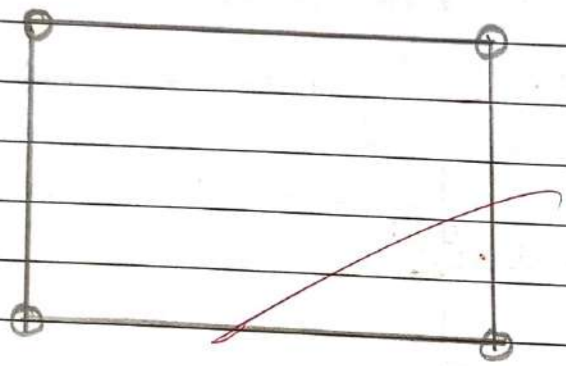
$$= \frac{3(3-1)}{2}$$

$$= \frac{3\cdot\cancel{2}^{\,1}}{\cancel{2}}$$

$$\boxed{=3}$$

For $\boxed{n=4}$

max Possible no. of edges $= \frac{n(n-1)}{2}$



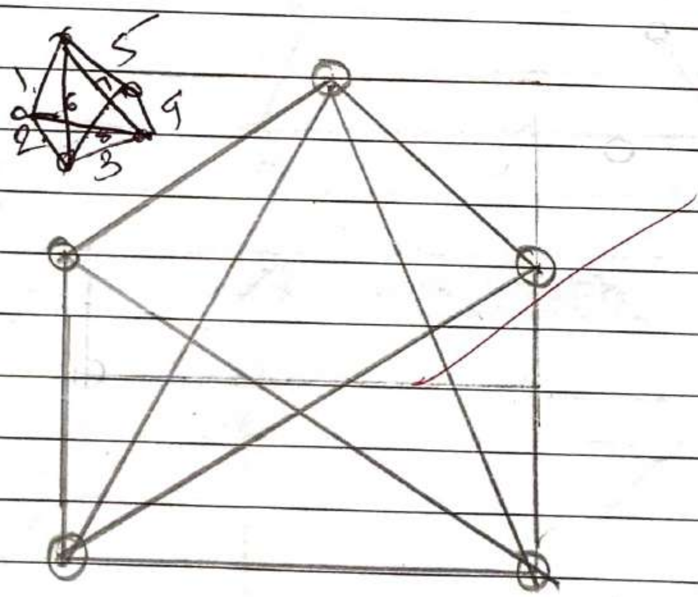
$= \frac{4(4-1)}{2}$

$= \frac{\cancel{4}^{2} \cdot 3}{\cancel{2}}$

$\boxed{=6}$

For $n=5$

Max Possible no. of edges $= \frac{n(n-1)}{2}$



$= \frac{5(5-1)}{2}$

$= \frac{5 \cdot 4 \cdot 2}{2}$

$\boxed{=10}$

2) For 'n' vertices the max possible no. graph Constructed is 2

Answer:— if $n=3$ then

PAGE NO.
DATE:

max. Possible no. of graph

$$= 2 \frac{(n-1)}{2}$$

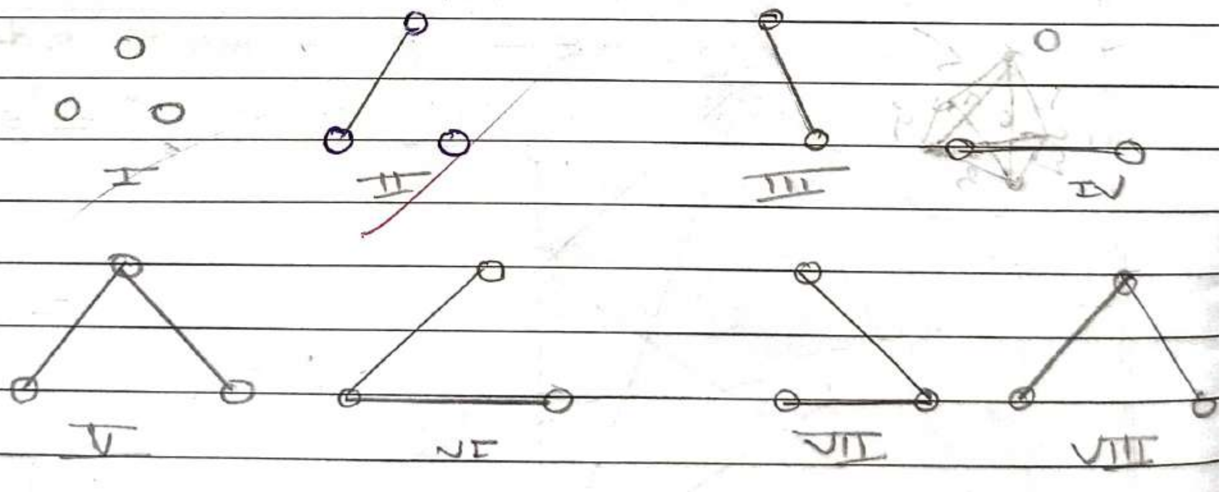
$$= 2 \frac{3(3-1)}{2}$$

$$= 3 \cdot \frac{2}{2}^{1}$$

$$= 2^{3}$$

$$\boxed{= 8}$$

Graph

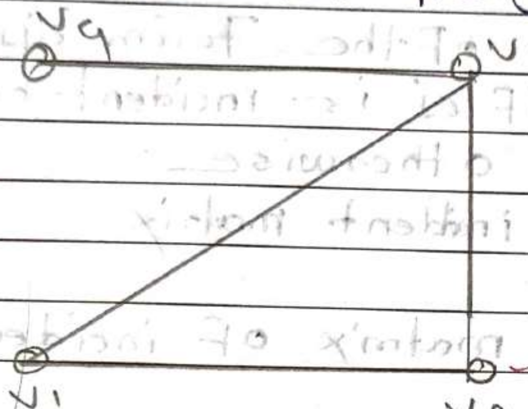


9 Subgraph and Induced graph

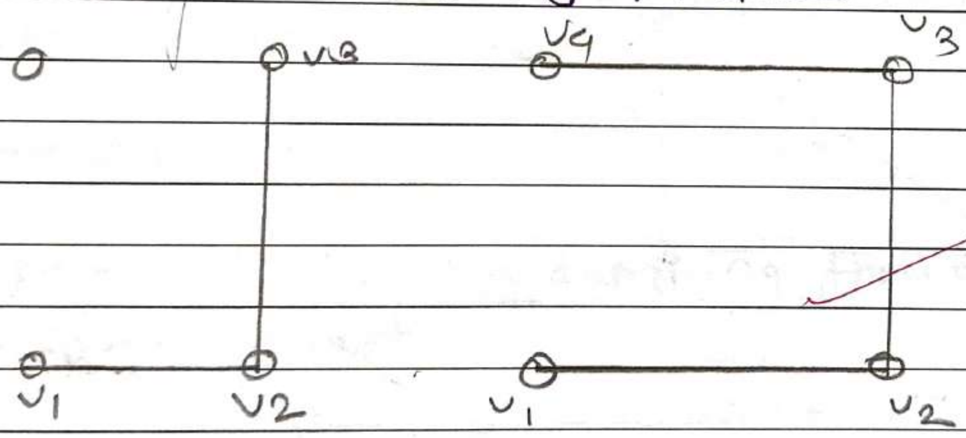
if G = (V, E) is a given graph then H = (V, E) is a said to be a subgraph

if $V_1 \subseteq V$ and $E \subseteq E$
it is said to be induced graph it is preserved adiencence.

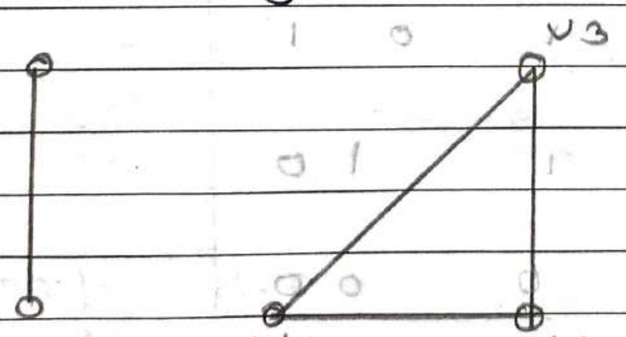
example - consider the graph.



Therefore, Subgraph are



induced subgraph.



10) Adajacent matrix and incident matrix

The matrix of the form a_{ij} =

a_{ij} 1 if v_i and v_j adjacent 0 otherwise is known as adjacent matrix

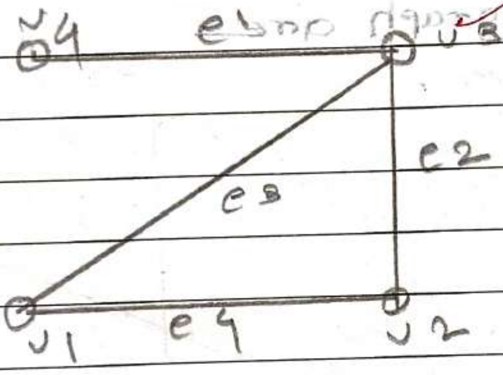
while,

The matrix of the form a_{ij}

$$a_{ij} = \begin{cases} 1 & \text{if } e_j \text{ is incident on } v_i \\ 0 & \text{otherwise} \end{cases}$$

is known incident matrix

example - Find matrix of incidence
Adjancy



0	0	1	1
0	1	0	1
1	1	1	0
1	0	0	0

The matrix of incidence is given belo above.

PAGE NO.
DATE:

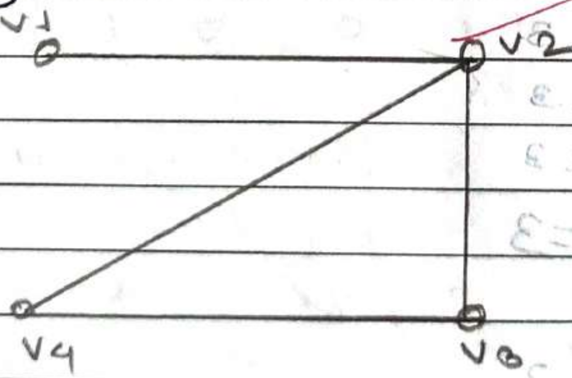
The matrix of Adjacent is given as,

	v_1	v_2	v_3	v_4
v_1	0	1	1	0
v_2	1	0	1	0
v_3	1	1	0	1
v_4	0	0	1	0

* Handshaking Theorem

example.

i) Verify the Handshaking theorem for a given graph.



Solution :- Here,

deg $(v_1) = 1$
deg $(v_2) = 3$
deg $(v_3) = 2$
deg $(v_4) = 2$

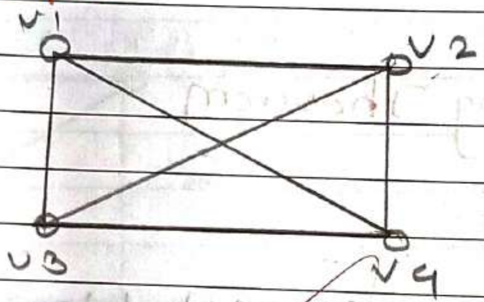
PAGE NO.
DATE:

Now,

$\deg(v_1) + \deg(v_2) + \deg(v_3) + \deg(v_4)$
$= 1 + 3 + 2 + 2$
$= 8$
$= 2 \times 4$

Therefore there four edges in a given graph Hence Handshaking theorem is verified

ii) Verify the Handshaking theorem for given graph.



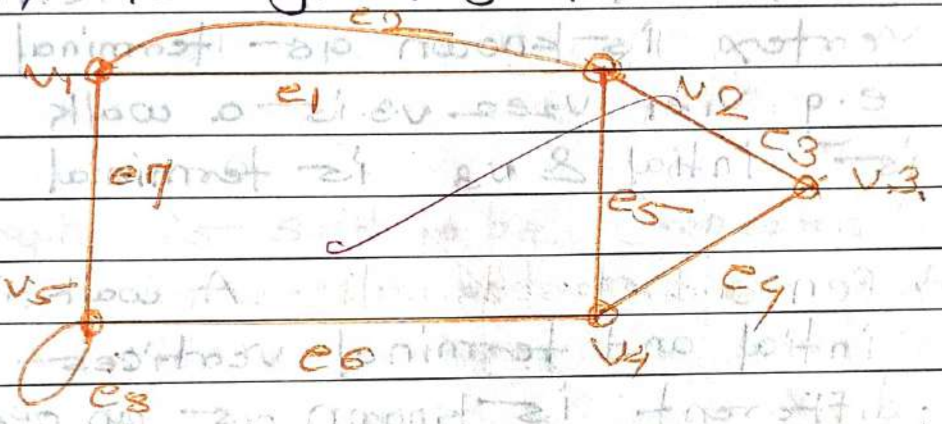
Here,
$\deg(v_1) = 3$
$\deg(v_2) = 3$
$\deg(v_3) = 3$
$\deg(v_4) = 3$
Now,
Consider,

$\deg(v_1) + \deg(v_2) + \deg(v_3) + \deg(v_4)$
$= 3 + 3 + 3 + 3$
$= 12$
$= 2 \times 6$

Therefore these are six edges

in the given graph
Hence,
Handshaking theorem is verified.

10) example (ii)
Find the adjacent matrix and incident matrix for a given graph.



	v_1	v_2	v_3	v_4	v_5
v_1	0	1	0	0	
v_2	1	0	1	1	0
v_3	0	1	0	1	0
v_4	0	1	1	0	1
v_5	1	0	0	1	1

incident matrix of the given graph

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
v_1	1	1	0	0	0	0	1	0
v_2	1	1	1	0	1	0	0	0
v_3	0	0	1	1	1	1	0	0
v_4	0	0	0	1	1	1	0	0
v_5	0	0	0	0	0	1	1	2

PAGE NO.
DATE:

11) Connectedness

a) walk - A Finite and alternating sequence of vertices of edges starting from a vertex and ends at a vertex is known as a walk. A starting vertex is known as intial vertex and end vertex is known as terminal vertex.
e.g $v_1 e_1 v_2 e_2 v_3$ is a walk in which v_1 is intial & v_3 is terminial vertex.

b) Open and closed walk - A walk in which intial and terminal vertices are different is known as an open walk while, a walk in which intial and terminal vertices are some is known as close walk.
eg - $v_1 e_1 v_2 e_2 v_3 e_3 v_4$ is a open walk
$v_1 e_2 v_2 e_2 v_3 e_3 v_1$ is a closed walk

c) Trial and path - An open walk in which edges are not repeated known as Trial while an open walk in which vertices are not repeated are known as path.

eg. $v_1 e_1 e_2 v_1 e_3 v_4$ is not a path
$e_1 v_1 e_2 v_2 v_3 e_3$ is a path

d) Circuit and Cycle - A closed walk in which edges are not repeated is known as Circuit. While, a closed

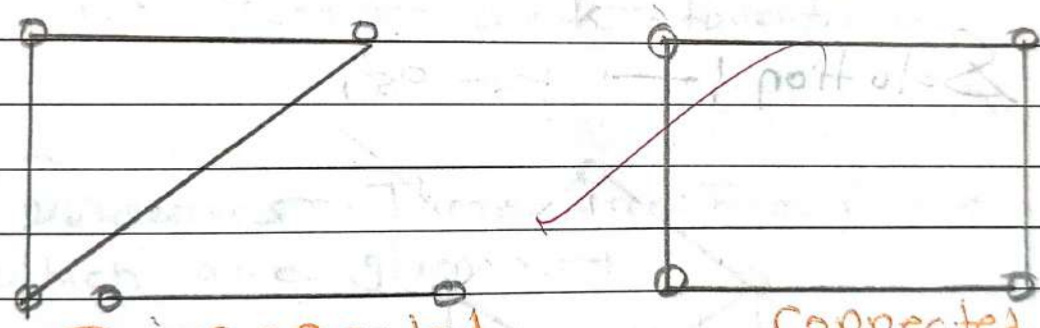
PAGE NO.
DATE:

walk in which vertices are not repeated except ~~walk in which vertices are not re~~ the intial vertex is called Called a cycle

example :— $v_1 e_2 v_3 e_3 v_4 e_5 v_1$ → circuit
$v_1 e_2 v_3 e_2 v_4 e_4 v_1$ → is a cycle

c) Connected & disconnected graph

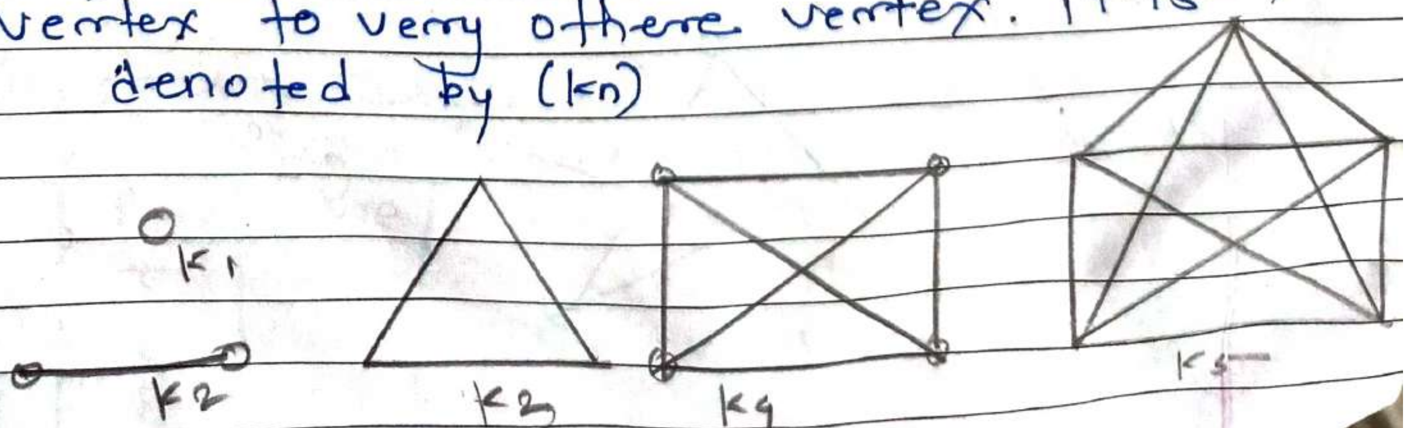
A graph is said to be Connected if there is a path between every pair of vertices otherwise ; disconnected.



Disconnected Connected

12) Complete graph and Compliment graph :-

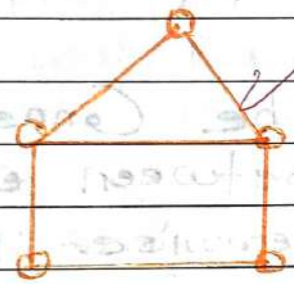
A graph is said to be a complete if there is an edge from every vertex to very othere vertex. it is denoted by (K_n)



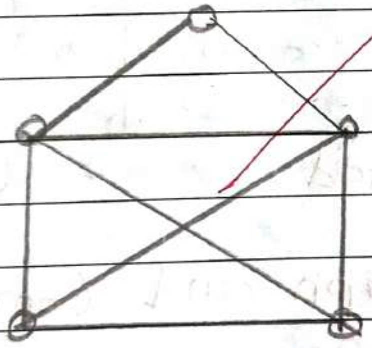
b. Complement of a graph

let a $G = (v,e)$ be a graph then the graph $K_n - G = \bar{G}$ is known as a Complement of G.

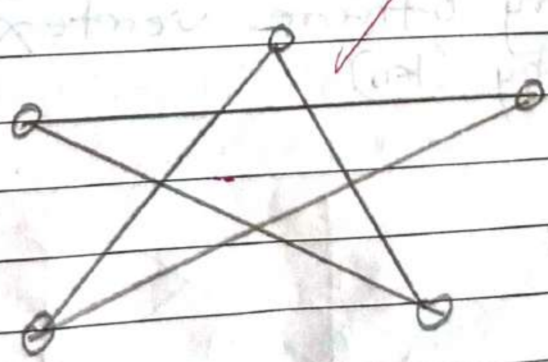
eg.
Find the Complement of given.



Solution :— K_5 as,



Now
$K_5 - G$ is given as —



Excellent
05/03/24

26/02/2024 Monday

PAGE NO.
DATE:

MATHEMATICAL LOGIC

① Propositional logical & Connectives

a) Propositional

The statement or assertion whose answer is Yes or No but not both at a time is known as a proportional

i) Pune is a Capital of India
ii) 2 + 2 = 4
iii) 5 > 2 and 2 > 5
iv) John is weak student

b. Connectives

There are Five Connectives which are given as

i) NOT = Negation - ~/¬
ii) and = Conjunction - ∧
iii) or = disjunction - ∨
iv) if and then = Conditional - →
v) if and only if = Biconditional - ↔

2) Truth Table

p	~p
T	F
F	T

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

Rule - when both are True resultant is - True otherwise False.

c. Truth table for disJunction

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

rule -
when both are False resultant is False, otherwise True

D. Truth table for Conditional

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

rule - when First is True another is False, resultant is False otherwise True

e. Truth table for BioConditional

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

rule — when both are same resultant is True otherwise False.

Problems

i) Construct the truth Table. For $\neg p \wedge q$

p	q	$\neg p \wedge q$	$\neg p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

ii) Construct the truth table for $(p \vee q) \vee \neg p$

p	q	$p \vee q$	$\neg p$	$(p \vee q) \vee \neg p$
T	T	T	F	T
T	F	T	F	T
F	T	T	T	T
F	F	F	T	T

PAGE NO.
DATE:

Q) Construct the truth table of the compound proposition $(p \vee \neg q) \rightarrow (p \wedge q)$

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Q4) Construct a truth table for $(p \rightarrow q) \leftrightarrow (r \leftrightarrow s)$

Solution –

Excellent

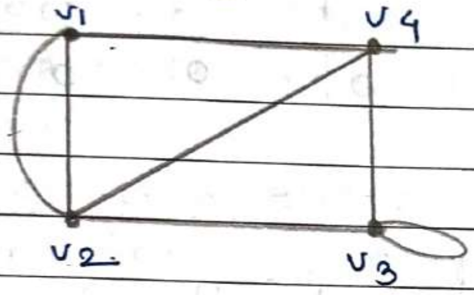
Checked 05/03/24

PAGE NO.
DATE:

4 March 2024 Monday

Assignment - 1

Q1) Verify the handshaking theorem.

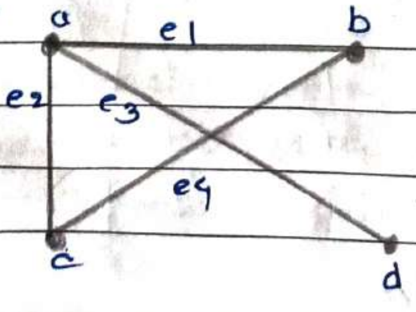


deg (v1) = 3
deg (v2) = 4
deg (v3) = 4
deg (v4) = 3

Now Consider,
deg (v1) + deg (v2) + deg (v3) + deg (v4)
= 3 + 4 + 4 + 3
= 14
= 2 x 7

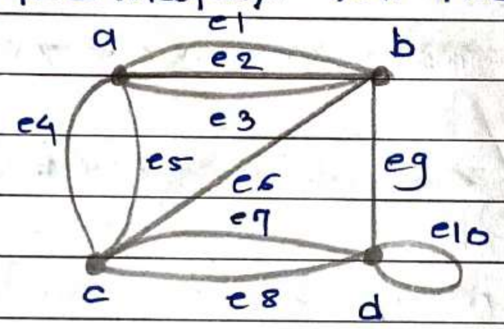
Therefore there seven edges in given graph
Hence Handshaking theorem is verified.

Q2) Find the adjacement matrix for the following graphs.



	a	b	c	d
a	0	1	1	1
b	0	0	1	0
c	1	1	0	0
d	1	0	0	0

Q 3} Find the matrix for the given graph.



	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10
a	1	1	1	1	1	0	0	0	0	0
b	1	1	1	0	0	1	0	0	1	0
c	0	0	0	1	1	1	1	1	0	0
d	0	0	0	0	0	0	1	1	1	1

PAGE NO.
DATE:

Q4} Explain the terms walk open and closed walk, trial, path, Circuit and Cycle with appropriate example.

a} walk :— A Finite and alternating Sequence of Vertices of edges starting From a vertex and ends at a vertex known as walk. A starting vertex is known as intial vertex. and end vertex is known as a terminal vertex.

e.g $v_1 e_1, v_2 e_2, v_3$ is a walk in which v_1 is intial & v_3 is terminal vertex.

b} open and closed walk :— A walk in which intial and terminal vertex is known as open walk, while a walk in which in which intial and terminal vertex is same is known as closed walk.

eg. $v_1 e_1$ $v_2 e_2$ $v_3 e_3$ v_4 is a open walk

$v_1 e_2$, $v_2 e_2$, $v_3 e_3$ v_1 is a closed walk

c} Trial and path :— A walk is open walk or the ages are not repeated is known as Trial. A walk is open walk or the vertices are not repeated is known as path.

eg. $v_1 e_1$, $e_2 v_1$, $e_3 v_4$ is

$e_1 v_1$, $e_2 v_3$, $v_3 e_3$. is a path

d} Circuit and Cycle :— A walk is closed walk the ages are not repeated is known as Circuit. A walk is closed walk

PAGE NO.
DATE:

while walk is closed walk or the vertices are not repeated except the intial Vertex is known as Cycle.

Excellent

eg $v_1 e_2 \; v_3 e_3 \; v_4 e_5$ is Circuit.
$v_1 e_2 \; v_3 e_2 \; v_4 e_4$ is Cycle.

Checked
05

- Converse, Contrapositive, AND Inverse

if $p \to q$ is a Conditional statement, then

1) $q \to p$ is Called its Converse
2) $\neg p \to \neg q$ is Called it's InversE
3) $\neg q \to \neg p$ is Called it's Contrapositive.

- Propositional logic equivalences

Tautology
A Compound Prepositiona that is always true, no matter what the truth values of the propositional variables that Occur in it, is called a tautology.

example :- $p \vee \neg p$

Contradiction
A Compound prepositional that is always False is Called a Contradiction.
example :- $p \wedge \neg p$

Contingency

A Compound Preposition that neither a tautology nor a Contradiction is a Called a Contingency.

logically equivalent

The Compound Prepositions p and q are Called logically equivalent if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ or $p \leftrightarrow q$ denotes that p and q are logically equivalent.

Problems

① show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent by by using truth table

Solution:-

Truth tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Q2) show that $p \to q$ and $\neg p \vee q$ are logically equivalent by using truth table.

p	q	$\sim p$	$\neg p \vee q$	$p \to q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\therefore p \to q \equiv \neg p \vee q$

Q3) Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$

PAGE NO.
DATE:

Assignment - 2

1} what is a simple graph? Give some example?

→ The Graph which does not Contain multiple edges is Called as simple graph.

Q 2} what is order and size of Graph? Define null graph and trivial graph

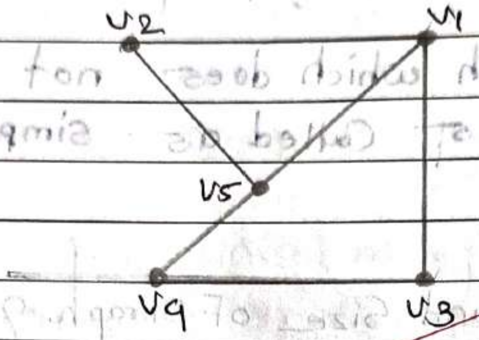
→ The Order of graph is its number |V| of vertices, usually denoted by n. The size of a graph is its number |E| of edges, typically denoted by m.

• Null graph The graph is said to be null graph, if it has any number of vertices without edges.

• Trivial graph :— The graph is said to be trivial Graph if it has a single vertex without edges.

Q] Define neighborhood and degree of degree.

neighborhood of vertices 'v' is the set of all the which are adjacent to 'v'.

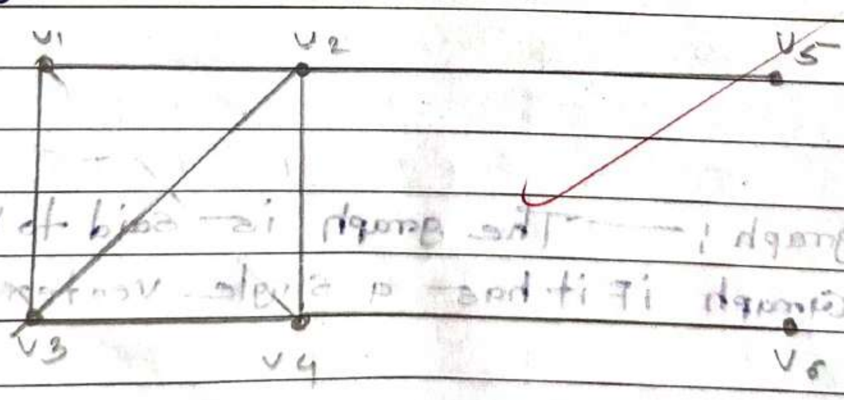


Degree

nhd of v_1 = {v_2, v_4}
nhd of v_2 = {v_5, v_1}
nhd of v_3 = { }

- Degree of vertices

The Degree of vertices 'v' is the no of edges incident on 'v'



deg(v_1) = 2
deg(v_2) = 4
deg(v_3) = 1
deg(v_4) = 3

deg(v_5) = 3
deg(v_6) = 1
deg(v_7) = 0

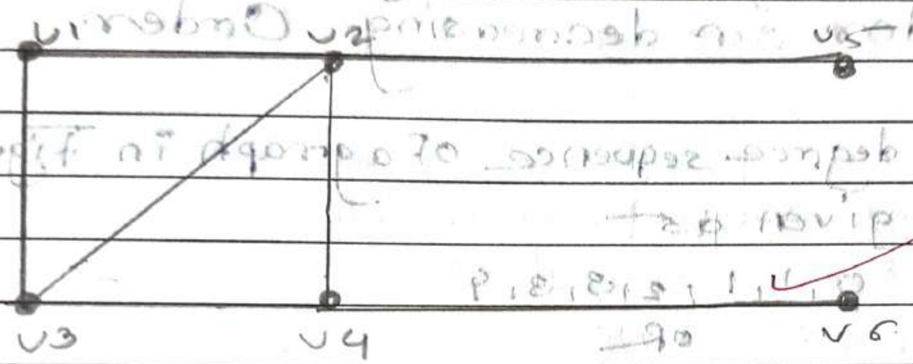
PAGE NO.

DATE:

Q4) What are Pendent and Isolated vertices? Give so example?

The vertex 'v' is said to Pendent vertex if it is '1'.

In the figure, 'v3' and 'v6' are Pendent vertices.



deg (v1) = 2

deg (v2) = 4

deg (v3) = 1

deg (v4) = 3

deg (v5) = 3

deg (v6) = 1

deg (v7) = 0

deg (v8) =

In Figure 'v3 and v6' are Pendent

The vertex 'v' is said to be Isolated vertex if it is degree is '0'.

In Figure 1 'v7' is an Isolated vertex.

PAGE NO.
DATE:

Q 5) what degree sequence of graph ? state. Handshaking theorem.

The degree sequence of a graph is the Collection of degree of all the vertices of the graph arrange in either increasin Order or decreasing Order

The degree sequence of a graph in Figure 1. is given as

0, 1, 1, 2, 3, 3, 4

OR

4, 3, 3, 2, 1, 1, 0.

Handshaking theorem

it's state's that the Sum of degree of all the vertices of a graph is twice the number of edges.

in Figure 1 we have

0 + 1 + 1 + 2 + 3 + 3 + 4 = 14 = 2×7

where '7' is the number of edges

Hence, Handshaking thorem is verified for the graph in Figure 1.

Mathematically we have

$$\sum_{v \in V} deg(v) = 2m$$

where 'm' is number of edge

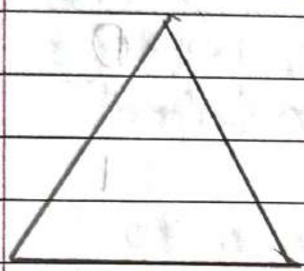
PAGE NO.
DATE:

① 6) Construct the maximum possible number of edges with the given vertices $n = 3, 4, 5$.

For $n=3$
Max Possible no. of edges-

$$\text{edges} = \frac{n(n-1)}{2}$$

$$= \frac{3(3-1)}{2}$$



$$= \frac{3 \cdot 2}{2}$$

$$\boxed{=3}$$

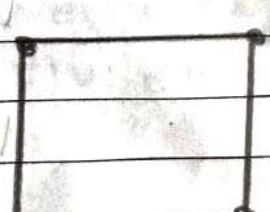
For $n=4$
Max Possible no. of edges-

$$\text{edges} = \frac{n(n-1)}{2}$$

$$= \frac{4(4-1)}{2}$$

$$= \frac{\overset{2}{\cancel{4}} \cdot 3}{\cancel{2}}$$

$$\boxed{=6}$$



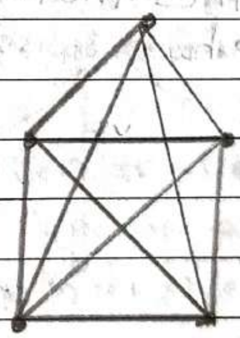
For $n=5$
Max Possible edges-

$$= \frac{n(n-1)}{2}$$

$$= \frac{5(5-1)}{2}$$

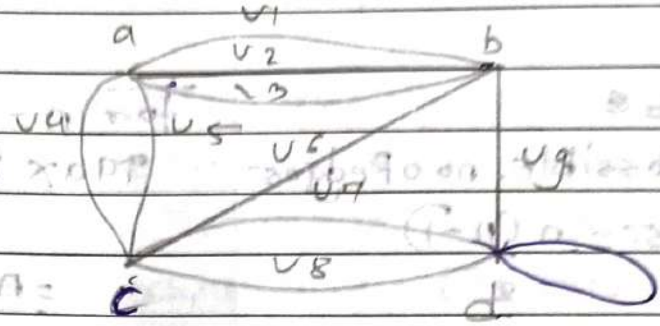
$$= \frac{5 \cdot \overset{2}{\cancel{4}}}{\cancel{2}}$$

$$\boxed{=10}$$



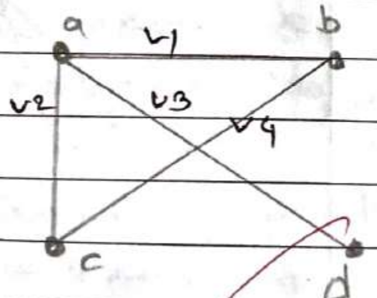
PAGE NO.
DATE:

Q7} Find the adjacent matrix for the following graphs.



	a	b	c	d
a	0	1	1	0
b	1	0	1	1
c	1	1	0	1
d	0	1	1	0

Q8} Find the matrix of the incidence for the graph.



	v1	v2	v3	v4
a	1	1	1	0
b	1	0	0	1
c	0	1	0	1
d	0	0	1	0

PAGE NO.
DATE:

Q9} Explain the term walk, open and closed walk, trial, path, Circuit and Cycle with appropriate examples.

a. walk A Finite and alternating Sequence of vertices and edges starting Form a vertex and ends at a vertex is known as a walk. A starting Vertex is known as intial vertex and end Vertex is known as terminial vertex.
eg $v_1 e_1, v_2 e_2$ is a walk in which v_1 is a intial and v_3 is terminal vertex.

b} open walk and closed walk :- A walk in which intial and terminal vertices are different is known as an Open walk. while a walk in which intial and terminal vertice are Same is known as close walk.
eg. $v_1 e_1, v_2 e_2, v_3 e_3, v_4$ is open walk.
$v_1 e_1, v_2 e_2, v_3 e_3\ v_1$ is a closed walk.

c} Trial and path :- An open walk in which edges are not repeated known as Trial. while an open walk in which vertices are not repeated are known as Path.
eg. $v_1 e_1, v_2 e_2, v_3 e_1, v_4$ not a trial.
$v_1 e_1, v_2, e_2\ v_3$ is a Trial.
Also,
$v_1 e_1, v_2 e_2, v_1 e_3\ v_4$ is a not a path.
$e_1 v_1\ e_2 v_2\ v_3 e_3$ is a Path.

PAGE NO.
DATE:

d) Circuit and Cycle – A closed walk in which edges are not repeated is known as Circuit. while a closed walk in which vertices are not repeated except the initial vertex is called a Cycle.

example:– $v_1 e_2, v_3 e_3, v_4 e_5 v_1$ → Circuit
$v_1 e_2 v_3 e_2 v_4 e_4 v_1$ → is a Cycle

Q10) Define Preposition with some appropriate examples
The statement or assertion whose answer is yes or No but not both at a time is a known as Prepositional

ex:— i) pune is a Capital of india
ii) $2+2=4$
iii) $5>2$ and $2>5$
iv) John is a weak student (invalid)

Q11) what are different type of Connectives and draw the truth table for the same

There are Five Connectives which are given as:—

1) Not – Negation

PAGE NO.
DATE:

2) and - Conjunction
3) ·Or - disjunction
4) if and then - Conditional
5) if and only if - BioConditional.
6

- Truth table

a) Truth table For Negation

p	~p
T	F
F	T

b) Truth table For Conjunction

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

Rule - when both are True resultant is true otherwise False.

c) Truth table For disjunction.

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

Rule :- when both are False resultant is otherwise True.

PAGE NO.
DATE:

2) and – Conjunction
3) or – disjunction
4) if and then – Conditional
5) if and only if – BioConditional
6

- Truth table

a) Truth table For Negation

p	~p
T	F
F	T

b) Truth table For Conjunction

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

Rule – when both are True resultant is true otherwise False.

c) Truth table For disjunction

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

Rule :– when both are False resultant is otherwise True.

PAGE NO.
DATE:

d. Truth table For Conditional

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

- Rule:- when First is True and other is False, resultant is False otherwise True.

e. Truth table For BioConditional

p	q	p ↔ q
T	T	T
T	F	F
F	T	F
F	F	T

Rule:— when both are same resultant is true, otherwise False.

Q 12} Construct the truth table for $\neg p \wedge q$

p	q	$\neg p$	$\neg p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

Q 13} Construct the truth table for $(p \vee q) \vee \neg p$

p	q	$p \vee q$	$\neg p$	$(p \vee q) \vee \neg p$
T	T	T	F	T
T	F	T	F	T
F	T	T	T	T
F	F	F	T	T

Q 14} Construct the truth table for the Compound Preposition $(p \vee \neg q) \rightarrow (p \wedge q)$

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

PAGE NO.
DATE:

Q15} Construct a truth table for $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$

p	q	r	s	$p \leftrightarrow q$	$r \leftrightarrow s$	$(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$
T	T	T	T	T	T	T
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	T	F	F	T	T	T
T	F	T	T	F	T	F
T	F	T	F	F	F	T
T	F	F	T	F	F	T
T	F	F	F	F	T	F
F	T	T	T	F	T	F
F	T	T	F	F	F	T
F	T	F	T	F	F	T
F	T	F	F	F	T	F
F	F	T	T	T	T	T
F	F	T	F	T	F	F
F	F	F	T	T	F	F
F	F	F	F	T	T	T

Q16} Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent by using truth table.

Truth table for $\neg(p \vee q)$ and $\neg p \wedge \neg q$

PAGE NO.
DATE:

Q15) Construct a truth table for $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$

p	q	r	s	$p \leftrightarrow q$	$r \leftrightarrow s$	$(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$
T	T	T	T	T	T	T
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	T	F	F	T	T	T
T	F	T	T	F	T	F
T	F	T	F	F	F	T
T	F	F	T	F	F	T
T	F	F	F	F	T	F
F	T	T	T	F	T	F
F	T	T	F	F	F	T
F	T	F	T	F	F	T
F	T	F	F	F	T	F
F	F	T	T	T	T	T
F	F	T	F	T	F	F
F	F	F	T	T	F	F
F	F	F	F	T	T	T

Q16) Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent by using truth table.

Truth table for $\neg(p \vee q)$ and $\neg p \wedge \neg q$

PAGE NO.
DATE:

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

$\therefore \neg(p \vee q) \equiv \neg p \wedge \neg q$

Q17} Show that $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent by using truth table.

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\therefore p \rightarrow q \equiv \neg p \vee q$

Q18} Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

p	q	r	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T						
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	F	T	T	T	T
F	F	F	T	T	T	T
F	T	T	T	T	T	T
F	T	F	F	T	F	F
F	F	T	F	F	T	F
	F	F	F	F	F	F

$\therefore p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Q 19) Show that the preposition $(p \vee q) \leftrightarrow (q \vee p)$ is tautology

p	q	$p \vee q$	$q \vee p$	$(p \vee q) \leftrightarrow (q \vee p)$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

the last Column entities are T
$\therefore (p \vee q) \longleftrightarrow (q \vee p)$ is tautology.

PAGE NO.
DATE:

20) obtain DNF of $p \wedge (p \rightarrow q)$

$p \wedge (p \rightarrow q) \equiv p \wedge (\neg p \vee q)$ (by logical equivalence involving condition)

$\equiv (p \wedge \neg p) \vee (p \wedge q)$ (By Distributive)

which is the required DNF.

Q21) obtain the DNF $p \rightarrow [(p \rightarrow q) \wedge \neg(\neg q \vee p)]$

Solution :—

(by logical equivalance involving condition)

$\equiv \neg p \vee [(\neg p \vee q) \wedge \neg(\neg q \vee \neg p)]$ (by double

$\equiv \neg p \vee [(\neg p \vee q) \wedge (q \wedge p)]$ (distributive law)

$\equiv \neg p \vee [(\neg p \wedge q \wedge p) \vee (q \wedge q \wedge p)]$ (negation law,

$\equiv \neg p \vee [((F \wedge q) \vee (q \wedge p)]$ idempotent

$\equiv \neg p \vee [(F \vee (q \wedge p)]$ law)

(domination law)

$\equiv \neg p \vee (q \wedge p)$ (Identity law)

which is the required DNF

VM Excellent

Checked
28/3/24

PAGE NO.

DATE:

MODULE - 3

Algebraic Structure.

- Set theory and Function

Cartesian Product

$A = \{1, 2\}, \quad B = \{a, b, c\}$

- Set - Roster and
- Power set
- Cartesian Product.
-
- Function
- 1R1 → Reflexive

  1R2 and 2R1

  symmetric

aRb and bRc → aRc

Transitive relation

15 April 2024 Monday

ASSINGMENT -3

- Sets :—

- DEFINITION :
A set is an unordered Collection

- Definition :—
the objects in a set are Called elements or members of the set. A set is a said Contain its elements. we write $a \in A$ to Denotes that is not an element of the set A.

- Example :—

  - The set V of all vowels in the english alphabet Can be written as V = {a, e, i, o, u}

  - The set O of odd positive integers less than 10 be expressed by O = {1, 3, 5, 7, 9}

- Equal sets :—

Two sets are equal only if they have same elements. therefore, if A, and B are sets then A and B are equal if and only if $\forall x\ (x \in A \leftrightarrow x \in B)$. we write A = B if A and B are equal sets.

- Empty Sets :—

~~Two sets are~~.
There is a special set that has no elements. This set is called the empty or null set, and is denoted by $\emptyset$.
The empty set can also be denoted by $\{\}$.

- NOTE :—

- It Does not matter if an element of a set is listed more than once, so $\{1,3,3,3,5,5,5\}$ is the same as the set $\{1,3,5\}$ Because they have the same elements.

- the set with one element is called a Singleton set.

- Subset :—

The set A is a Subset of B if and only if every element of A is also and element of B. We use the notation.

$A \subseteq B$ to indicate that A is a subset of the set B.

showing that A is a subset of B. To show that $A \subseteq B$ show that if x belongs to A then x also belongs to B

- Example :-

- The set of all odd positive integers less than 10 is a subset of all the positive integers less than 10.

- The set of of rational numbers is a subset of the set of real numbers.

- Proper Subset :-

A set A is called Proper subset of the set B. if (i) A is subset of B and (ii) B is not subset A i.e. A is a proper subset B, than we donote, it by $A \subset B$.

- Cardinality :-

A set A is a called proper subset set B. if there are exactly n distinct elements in S is a finite set and that n is Cardinality of S. The Cardinality of S is denoted by $|S|$.

- Example :-

- let A be the set of odd positive integers less than 10. Then $|A| = 5$.

- let S be the set of letters in the english alphabet. then $|S| = 26$.

- INFINite :-
A set is said to be InFinite if it is not Finite.

- Example :-
the set of positive interger is infinite.

- Universal sets :-
The universal set U, which Contains all the objects under Consideration

- Power set :-
Given a set S, the Power set of S is the set of all subsets of the sets. the power set of S is denoted by P(S)
if a set has n elements, then its power set has of S is Denoted by

- Example :-
(1) what is the Power set of the set {0,1,2}

- Solution :-

The Power set P({0,1,2}) is a set of all Subsets of {0,1,2}

Note that the empty set and the set itself are members of this set subsets.

• (Cartesian Product :–

let the A and B be sets. the Cartesian product of A and B denoted by $A \times B$, is the set of all Ordered pairs (a, b), where $a \in A$ and $B \in B$. Hence, $A \times B = \{(a, b)\} \mid a \in A \wedge b \in B\}$

• Example :–

(1) what is the Cartesian Product of $A = \{1, 2\}$ and $B = \{a, b, c\}$?

Solution :– The Cartesian Product of $A = \{1, 2\}$ and $B = \{(1, a), (1, b), (1, c) (2, a). (2, b), (2, c)\}$

• Practice Problems :

(1) List the members of these sets.

(a) $\{x \mid x$ is a real number such that $x^2 = 1\}$

$A = \{1, -1\}$;

(b) $\{x \mid x$ is a positive numbers less 12$\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$;

c) $\{x \mid x$ is the square of an integer and $x < 1000\}$

$C = \{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$

Q2} Use set builder notation to give a description of each of these sets.

a) $\{0, 3, 6, 9, 12\}$

$= \{3n / n = 0, 1, 2, 3, 4\}$;

b) $\{-3, -2, -1, 0, 1, 2, 3\}$

$= \{x / -3 \le x \le 3\}$

Q3} What is the cardinality of each of these sets?

a} $\emptyset = 0$;

b} $\{\emptyset\} = 1$;

c} $\{\emptyset, \{\emptyset\}\} = 2$;

d} $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\} = 3$;

Q4} Find the power set of each of these sets, where a and b are distinct elements.

a) $\{a\}$

$= \{\emptyset \{a\}\}$;

b} $\{a, b\}$

$= \{\emptyset \{a\}, \{b\}, \{a, b\}\}$;

c} $\{\emptyset; \{\emptyset\}\}$

$= \{\emptyset \{0\} \circ \{\{0\}\} \circ \{0 \{0\}\}\}$

~~d} $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$~~

Q5} Find A^2 if

a} $A = \{0, 1, 3\}$;

$A = \{(0,0), (0,1), (0,3), (1,0), (1,3), (3,1), (3,3)\}$

b} $A = \{1, 2, a, b\}$

$A = \{(1,1), (1,2), (1,a), (1,b), (2,1), (2,2), (2,a), (2,b), (a,1), (a,2), (a,a), (a,b), (b,1), (b,2), (b,a), (b,b)\}$

- Set operations :—

Example :—
let $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 4, 6, 8, 10\}$
Because $A \cap B = \emptyset$, A and B are Disjoint.

- Union :—

let A and B be sets. the union of the sets A and B, denoted by $A \cup B$, is this set that Contains those elements that are either A or in B, or in both, it is denoted by
$A \cup B = \{x \mid x \in A \vee x \in B\}$.

- Example :—

(1) The union of the sets of all Computer Science majors at your school and the set of all mathematics major at your school is the set of students at your school who are majoring either in mathematics or in Computer science.

- Example :—
- INTERSEction.

Example :-
let A and B be sets. $\{1, 3, 5\}$ and $\{1, 2, 3\}$ is the set $\{1, 3\}$; that is, $\{1, 3, 5\} \cap \{1, 2, 3\}$

$= \{1, 3\}$

· let A and B be sets. the Intersection of the sets, A and B, denoted by $A \cap B$ is the set Containing those elements in both A and B. it is Denoted by
$A \cap B = \{x | x \in A \wedge x \in B\}$

• Difference of sets :—

let A and B be sets. the difference of A and B. Denoted by $A - B$ is the set. Containing those elements that are in A. but not in B. the difference of A and B is also Called the Complement of B with respect to A. it is denoted by $A - B$
thus : $A - B = \{x / x \in A \text{ and } x \in B\}$.

Example :— the difference of $\{1, 3, 5\}$ and $\{1, 2, 3\}$ is a set $\{5\}$; that is $\{1, 3, 5\} - \{1, 2, 3\} = \{5\}$ this and $\{1, 2, 3\}$ and $\{1, 3, 5\}$ which is a set $\{2\}$

• Complement of A set :—

let U be the Universal set. the Complement of set A. Denoted by $\overline{A}$, it is Denoted by $\overline{A}$ thus

$A = \{x \in U / x \in A\}$

• Example :—

① let $A = \{a, e, i, o, u\}$ (where the universal set of letter of the English alphabet) then

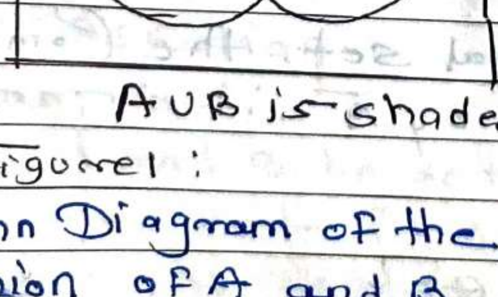
$A = \{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}$.

• Disjoints sets :—

Two sets are called disjoint if their intersection is the empty set.

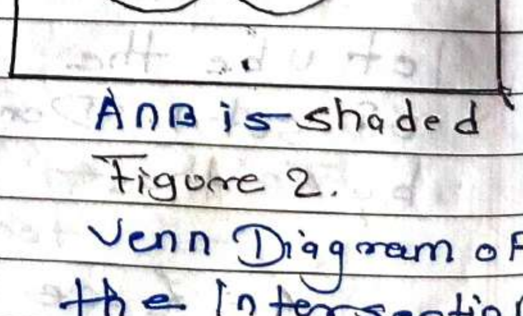
• Venn Diagram :—

A venn Diagram is a pictorial representation of the set. the universal set U is normally represented by a reactangle and its subsets as Circles inside it. Venn Diagramm are Useful in understanding relation among sets and operation on set



$A \cup B$ is shaded

Figure1 : Venn Diagram of the union of A and B



$A \cap B$ is shaded

Figure 2. Venn Diagram of the Intersection of A and B.

(1) let $A = \{a, e, i, o, u\}$ (where the universal set of letter of the English alphabet) then

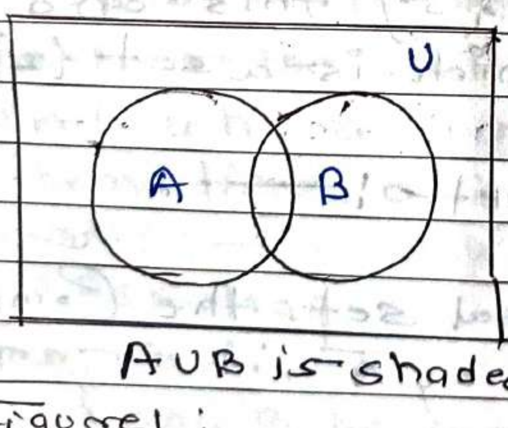
$A' = \{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}$.

• Disjoint sets :–

Two sets are called disjoint if their intersection is the empty set.

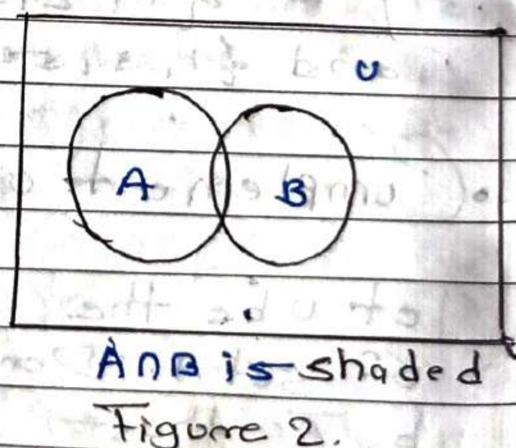
• Venn Diagram :–

A venn Diagram is a pictorial representation of the set. the universal set U is normally represented by a reactangle and its subsets as Circles inside it. Venn Diagramm are Useful in understanding relation among sets and operation on set



$A \cup B$ is shaded

Figure 1 :
Venn Diagram of the union of A and B



$A \cap B$ is shaded

Figure 2.
Venn Diagram of the Intersection of A and B.

- Functions :—

- Defination :—

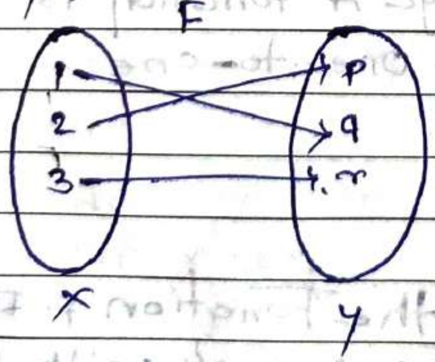
let A and B be nonempty sets. A Functions F From A to B is an assignment of exactly one element of B to each element of A. we write F(a) = b if b is the unique element of B assigned by the Fonction F to the elements of A. ~~We write F(a) = b if b is the on~~ IF F is a Function From A to B. we write $F: A \to B$.

- Note :—

Fonction are Sometimes also Called mapings or transformations.

- Example

let $X = \{1, 2, 3\}$, $Y = \{p, q, r\}$ and $F = \{(1, q), (2, p), (3, r)\}$ then $F(1) = q$, $F(2) = p$, $F3 = r$ clearly F is a Fonction From x to y.



- Example:—

(1) If the Function F is defined by $F(x) = x^2 + 1$ on the set $\{-2, -1, 0, 1, 2\}$. Find the range of F.

Solution:—

$F(-2) = (-2)^2 + 1 = 5$
$F(-1) = (-1)^2 + 1 = 2$
$F(0) = 0 + 1 = 1$
$F(1) = 1 + 1 = 2$
$F(2) = 4 + 1 = 5$

Therefore, the range of F = $\{1, 2, 5\}$

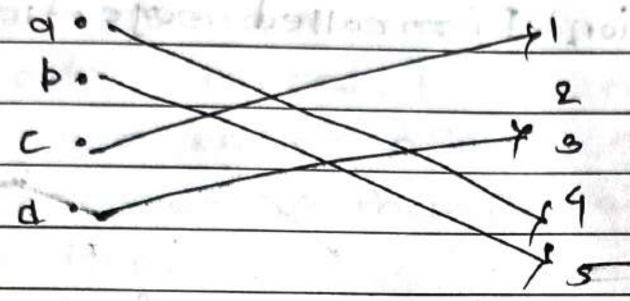
- One to One (Injective):—

A Function f is said to be one to one, or an injective, if and only if $F(a) = F(b)$ implies that $a = b$ for all a and b in the Domain of F. i.e. Distinct elements should have Distinct Image. A Function is said to be Injection if it is one-to-one.

- Example:—

Determine whether the Function F from $\{a, b, c, d\}$ to $\{1, 2, 3, 4, 5\}$ with $F(a) = 4$, $F(b) = 5$, $F(c) = 1$, and $F(d) = 3$ is one-to-one.

Solution; — The Function F is one-to-one Because F takes on different values at an Four elements of its domain.



2. Determine whether $F: Z \to Z$ given by $F(x)$ $F(x) = x^2, x \in Z$ is a one-to-one Function

Solution; — The Function $F: Z \to Z$ given by $F(x) = x^2, x \in Z$ is not a one-to-one Function. This is because both 3 and -3 have 9 as their image, which is against the defination of a One-to-one-Function.

3. Determine whether the Function $F(x) = x+1$ From the set of real numbers to itself is one-to-one solution

Solution; — The Function $F(x) = x+1$ is a one-to-one Function.

To demostrate this, note that $x+1 \neq y+1$ when $x \neq y$

- **Onto (Surjectives) :—**

A Function F from A to B is called onto, or a Surjective, if and only if for every element $b \in B$ there is an element $a \in A$ with $F(a) = b$: A Function F is called surjection if it is Onto.

Example :—

(1) Let F be the Function From {a, b, c, d} to {1, 2, 3} by F(a) = 3, F(b) = 2, F(c) = 1 and F(d) = 3 is F an onto Function.

Solution :—

All three elements of the Codomain are images of elements in the domain, we see that F is onto.

- **Bijective**

The Function F is a one-to-one Correspondence or a bijection, if it is both one-to-one and onto we also say that such a Function bijective.

- Example :—

(1) Let F be the Function from {a, b, c, d} to {1, 2, 3, 4} with F(a) = 4, F(b) = 2, F(c) = 1 and F(d) = 3, is F a bijection?

• Solution ;—

The function f is one-to-one and onto. It is one-to-one because no two values in the domain are assigned the same function value. It is onto because all four elements of the codomain are images of elements in the domain. Hence, f is a bijection.

• Relations and their Properties ;—

let A and B be sets. A binary relation from A to B is a subset of A×b.

In other words, a binary relation from A to B is a set R of ordered pairs when the first element of each ordered pair comes from A and the second elements comes from B. we use the notation aRb to denote that $(a,b) \in R$ and aRb to denote that $(a,b) \notin R$ Moreover, when (a,b) belongs to R, a is said to be realated to b by R.

• Defination ;—

A relation on a set A is a relation from A to A. In other words, a realation on a set A is a subset of A×A

• Example ;—

(1) Define a relation between two sets $A = \{5, 6, 7\}$ and $B = \{x, y\}$.

- Solution :— if $A = 5, 6, 7$ and $B = x, y$ then the subset $R = \{(5, x), (5, y), (6, x), (6, y)\}$ is a relation from A to B

- Properties of Relations :—

- A relation R on a set A is called reflexive if $(a, a) \in R$ for every element $a \in A$.

- A relation R on a set A is called symmetric if $(b, a) \in R$ whenever $(a, b) \in R$ for all $a, b \in A$.

- A relation R on set A such that for all $a, b \in A$, if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ is called antisymmetric.

- A relation R on set A is called transitive if whenever $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$ for all $a, b, c \in A$.

• Example :-

1. Consider the following relations on $\{1,2,3,4\}$:

• $R_1 = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)\}$,

• $R_2 = \{(1,1), (1,2), (2,1)\}$,

• $R_3 = \{(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (4,1), (4,4)\}$,

• $R_4 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\}$

• $R_5 = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\}$

• $R_6 = \{(3,4)\}$ which of these relations are reflexive, symmetric, antisymmetric and transitive?

• Solution :-

the relations R_3 and R_5 are reflexive because because they both contain all pairs of the from (a,b), namely $(1,1)$, $(2,2)$, $(3,3)$ and $(4,4)$. the other relations are not reflexive because they do not contain all of these ordered pairs. In particular R_1, R_2, R_3, R_4 and R_6

are not reflexive because (3,3) is not in any of these relation

the relation R2 and R3 are symmetric because in each case (b,a) belongs to the relation whenever (a,b) does. For R2, the only thing to check is that both (2,1) and (1,2) are in the relation. For R3 it is necessary to check that both (1,2) and (2,1) belong to the relation, and (1,4) and (4,1) belong to the relation.

R4, R5 and R6 are all antisymmetric. For each of these relations there is no pair of element a and b with $a \neq b$ such that both (a,b) and (b,a) belong to the relation

R4, R5 and R6 are transitive. For each of these relations, we can show that it is transitive by verifying that if (a,b) and (b,c) belong to this relation, then (a,c) also does. For instance, R4 is

transitive because (3,2) and (2,1), (4,2) and (2,1), (4,3), (3,1) and (4,3) and (3,2) are the only such sets of pairs, and (3,1), (4,1) and (4,2) belong to R4.

Excellent
Checked
19/04/24

Assignment-9

Module - 9
Relation and Partially ordering

- Matrix representation.

$A = \{a, b, c, d\}$
$B = \{1, 2, 3\}$
$R = \{(a, b), (a, 2), (c, 3), (d, 2)\}$

- Relation Matrix

A relation R From a Finite set x to a Finite set of y can be represented by a matrix is called the relation matrix of R let $A = \{a_1, a_2, a_3 . a_m\}$ and $B = \{b_1, b_2, b_n\}$ be Finite sets Containing m and n elements, respectively, and R be the relation From A to B. Then R can be represented by an $m \times n$ matrix $M_R = |m_{ij}|$, which is definied as Follows:

$$m_{ij} = \begin{cases} 1 \text{ if } (a_i, b_j) \in R. \\ 0 \text{ if } (a_i, b_j) \notin R. \end{cases}$$

Example:-

1. suppose that $A = \{1, 2, 3\}$ and $B = \{1, 2\}$ Let R be the Relation From A to B Containing (a, b) if $a \in A$, $b \in B$. and $a > b$. what is the matrix representing R if $a_1 = 1$, $a_2 = 2$. and $a_3 = 3$ and $b_1 = 1$ and $b_2 = 2$?

Solution ;—
Because $R = \{(2,1),(3,1),(3,2)\}$, the matrix of R is

$$M_R = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Then 1s in M_R show that the pairs (2,1) (3,1) and (3,2) belong to R. The 0s show that no others pairs belong to R

Q2} Let $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3, b_4, b_5\}$ which ordered pairs are in the relation R represented by the matrix M_R =

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Solution ;— Because R Consists of those orderd Pairs (a_i, b_j) with $m_{ij} = 1$, it Follows that
$R = \{(a_1, b_2), (a_2, b_1), (a_2, b_3), (a_2, b_4), (a_3, b_1), (a_3, b_3), (a_3, b_5)$

3} suppose that the relation R on a set is represented by the matrix $M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ is represented by the matrix.

Solution :— Because all the Diagonal elements of this matrix are equal to 1, R is reflexive, moreover, because M_R is symmetric it follows that R is symmetric. it is also easy to see that the R is not antisymmetric.

Q4) Find the matrix representing the relation R^2, where the matrix represented R is

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Solution :— The matrix for R^2 is

$$M_{R^2} = M_R^{[2]} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Q5) Let $R = \{(1,2), (3,4), (2,2)\}$ and $S = \{(4,2), (2,5), (3,1), (1,3)\}$. Find $R \circ S$, $S \circ R$, $R \circ (S \circ R)$, $(R \circ S) \circ R$, $R \circ R$, $S \circ S$, and $(R \circ R) \circ R$

Solution :— Given $R = \{(1,2), (3,4), (2,2)\}$ and $S = \{(4,2), (2,5), (3,1), (1,3)\}$.

$R \circ S = \{(1,5), (3,2), (2,5)\}$

$S \circ R = \{(4,2), (3,2), (1,4)\}$

$R \circ (S \circ R) = \{(3,2)\} = (R \circ S) \circ R$

$S \circ R = \{(4,2), (3,2), (1,4)\}$

$R \circ R = \{(1,2), (2,2)\}$

$S \circ S = \{(4,5), (3,3), (1,1)\}$

$(R \circ R) \circ R = \{(1,2), (2,2)\}$

Q6) Example: Let $A = \{a, b, c\}$, and R and S be relations on A whose matrices are as given below:—

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

- Find the Composite relations $R \circ S$, $S \circ R$, $R \circ R$, $S \circ S$ and their matrices

Solution;—

$R = \{ (a, a), (a, c), (b, a), (b, b), (\quad), (\quad) \}$

$S = \{$

$R \circ S = \{$

$S \circ R = \{$

$R \circ R = \{$

$S \circ S = \{$

The matrices of the above Composite relations are as given below -

$$M_{R \circ S} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}; \quad M_{S \circ R} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix};$$

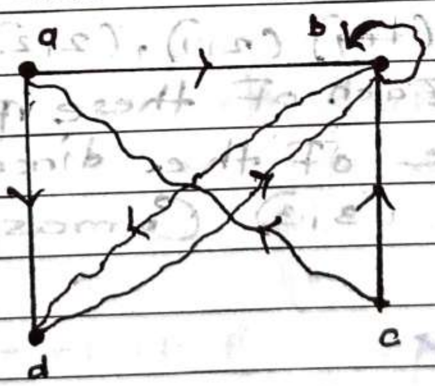
$$M_{R \circ R} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad M_{S \circ S} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Directed Graph —

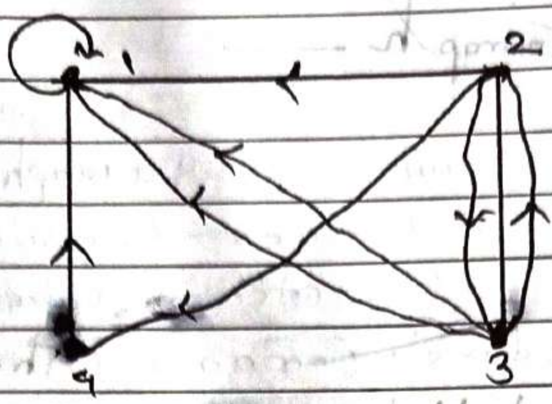
A Directed Graph, or Digraph, Consists of a set V of vertices (or nodes) together with a set E of ordered pairs of elements of V called edges (or arcs). The vertex a is called the intial vertex of the edge (a,b) and the vertex b is called the terminal vertex of this edge

Note :—
An edge of the form (a,a) is represented using an arc from the vertex a back to-itself such an edge is called a loop.

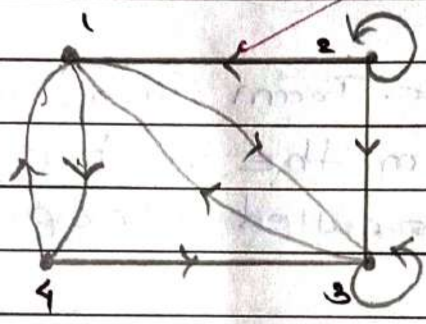
Example 1—
1) The Directed graph with vertices a, b, c and d, and edges (a,b), (a,d), (b,b), (b,d), (c,a), (c,b) and (d,b)



2) The Directed graph of the relation
R = {(1,1), (1,3), (2,1), (2,3), (2,4), (3,1), (3,2), (4,1)} on the set {1,2,3,4} is shown in



3) what are ordered pairs in the relation R respectively by the directed graph shown in



Solution : —

The ordered pairs (x, y) in the relation
$R = \{(1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (4,1), (4,3)\}$. Each of these pairs corresponds to an edge of the directed graph with $(2,2)$ and $(3,3)$ corresponding to loops

PAGE NO.

DATE:

Equivalence Relations :—

A relation on a set A is called an equivalence relation if it is reflextive, symmetric, and transitive. two elements a and b that related by an equivalent element with respect to a particular equivalence relation.

Example :—

1. let R be the relation on the set of real number that aRb if and only if a-b is an integer is R an equivalence relation ?

Solution :—

i) Because a-a=0 is an integer for all the real numbers, aRa for all real numbers a.

Hence, R is reflexative.

ii) Now suppose that aRb then a-b is an integer so b-a is also integer.

Hence bRa

it follows that R is symmetric.

iii) if aRb and bRc, then a-b and b-c are integer.

Hence, aRc.

Thus, R is transitive

Consequently, R is an equivalence relation

PAGE NO.
DATE:

9) Let $X = \{1, 2, 3 \ldots 7\}$ and $R = \{(x, y)\} x - y$ is divisible by $3\}$. show R is an equivalence relation.

Solution:—

i) For any $x \in X$ $x - x = 0$ is divisible by 3

$\therefore xRx$
$\Rightarrow$ R is reflexive.

ii) For any $x, y \in X$, if xRy, the $x - y$ is divisible by 3.

$\rightarrow -(x-y)$ is divisible by 3.
$y - x$ is divisible by 3
$\rightarrow yRx$

Thus, the relation R is symmetric

iii) For any $x, y, z \in X$ let xRy and yRz

$\rightarrow (x-y) + (y-z)$ is divisible by 3
$x - z$ is divisible by 3
xRz

Hence, the relation R is transitive
Thus, the relation R is an equivalence relation.

PAGE NO.
DATE:

Q3) Suppose that R is the relation on the set of strings of english letters such that aRb if and only if $l(a) = l(b)$ where $l(x)$ is the length of the strings x. is R an equience relation?

Solution:—

i) Because $l(a) = l(a)$ it follows that aRa whenever a is a string, so that R is reflexive

ii) Next, Suppose that aRb, so that $l(a) = l(b)$ then bRa because $l(b) = l(a)$.
Hence R is a symmetric.

iii) Finally, suppose that aRb and bRc. then $l(a) = l(b)$ and $l(b) = l(c)$. Hence $l(a) = l(c)$, so aRc Consequently, R is transitive.

Because R is reflexive, symmetric and transitive; it is an equilence a relation.

PAGE NO.
DATE:

- Partial Orderings

- A relation R on set S is called a Partial order relation if it is reflexive, antisymmetric and transitive. A set S together with a Partial ordering R is called a Partially Ordered set or poset, and is denoted by (S.R). Members of S are called elements of the poset.

- Example;—.

Let ≥ be the "greater than or equal". relation (≥) is a Partial Ordering on the set of integers Z.

Solution;—

Let Z be the set of all integers and the relation R = '≥'

Since a ≥ a for every integers a, the relation R = '≥'

i) Let a and b for every integer a, Let a R b and b R a → a ≥ b and b ≥ a = a = b

∴ the relation '≥' is antisymmetric

PAGE NO.
DATE:

iii) Let a, b and c be any three integers.
Let a R b and b R a ⇒ a ≥ b, a = a = b

∴ The relation '≥' is transitive. Since the relation '≥' is reflexive, anti-symmetric and transitive, '≥' is Partial Ordering on the set of integers.

therefore (Z, ≥) is a Poset.

Q3) The divisiblity relation | is a Partial Ordering on the set of positive integers.

Solution :—

let Z^+ be the set of Positive integers which doesn't include 0.

Since i) a/a For all $a \in Z^+$, | is reflexive

ii) a|b and b|a → a = b | is antisymmetric

iii) a|b and b|c → a|c, | is transverse

it Follows that | is Partial Ordering on Z^+ and $(Z^+, |)$ is a Poset.

PAGE NO.
DATE:

Hasse Diagrams.

A Partial Order $\leq$ on a set P can be represented by means of diagram known as Hasse diagram of P $(P,\leq)$,

in such diagram.

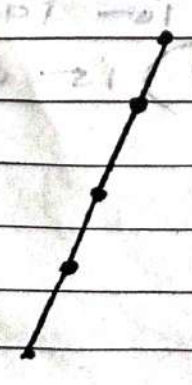
i) Each element is represented by a small circle or dot.

ii) The circle for $x \in P$ is drawn below the circle for $y \in P$ if $x < y$, and a line is drawn between x and y if y covers x.

iii) if $x \leq y$ but y does not cover x. then x and y are not connected directly by single line.

Example

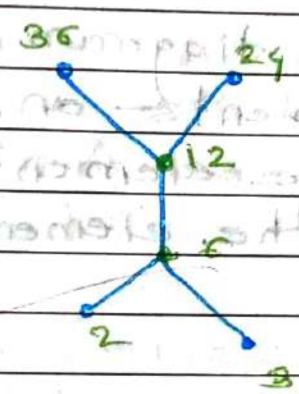
1) Let $P = \{1,2,3,4,5\}$ and $\leq$ be the relation "less than or equal to", then the Hasse diagram is



PAGE NO.
DATE:

(2) Let $X = \{2, 3, 6, 12, 24, 36\}$, and the relation $\leq$ be such that $x \leq y$ if x divides y. Draw the Hasse diagram of $(X, \leq)$

Solution :—

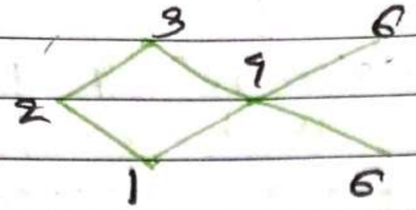


- Minimal And Maximal Element.

Let $(P, \leq)$ denote a partially ordered set. An element $y \in P$ is a called a minimal member of P relative to $\leq$ if for no $x \in P$ is $x \leq y$, similarly an element $y \in P$ is called a Maximal member of P relative to the partial ordering $\leq$ if for no $x \in P$, is $y \leq x$

Example :—

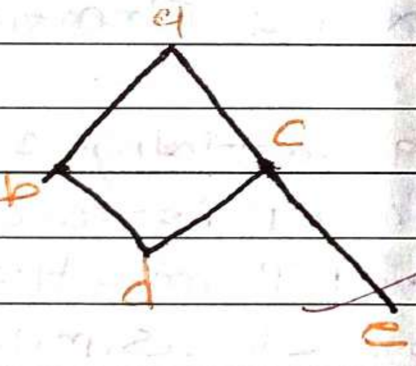
(1) Find the maximal and minimal elements.



Solution; →

in the Hasse diagram, there are two maximal elements and two minimal elements. the elements 3, 5 are maximal and the elements 1 to 8 are minimal.

(2) Find the maximal and minimal elements OF



let A = {a, b, c, d, e} and let the Partial Order on A in the natural way. the element a is maximal the elements d and e are minimal.

PAGE NO.
DATE:

- **Lexicographic Ordering**

the lexicographic ordering $\leq$ on, $A_1 \times A_2$ is defined by specifying the one pair is less than (in A_1) the first entry of the second pair, or if the first entries are equal, but the second entry of this pair is less than (in A_2) the second entry of the second pair.

In words (a_1, a_2) is less than (b_1, b_2) that is $(a_1, a_2) < (b_1, b_2)$. either if $(a_1 < b_1$ or if both $a_1 = b_1$ and $a_2 < 2 b_2$. we obtain a partial ordering by adding equality. to the ordering $<$ on $A_1 \times A_2$

Example:-

1) Determine whether $(3,5) < (4,8)$ whether $(3,8) < (4,5)$ and whether $(4,9) < (4,11)$ in the poset $(Z \times Z)$, where is the lexicographic ordering constructed from the usual $\leq$ relation on Z.

Solution:-

PAGE NO.
DATE:

Because $3<4$ it follows that $(3,5)<(4,8)$ and that $(3,8)<(4,5)$
we have $(4,9)<(4,11)$
because the first entries of $(4,9)$ and $(4,11)$ are same
but $9<11$

• 10/05/2024. Friday Assignment - 5.

5. probability

Basic probability Concepts :-

Random Experiment :-

An activity that produces an outcome is referred to as a experiment is said to be a random experiment if its outcome cannot be predicted with certainity.

eg.
If a coin is tossed, one cannot predict beforehand whether head or tail will appear so, it is a random experiment.

• Sample space :-
The set of all possible outcomes of a experiment is called the sample space it is denoted by 'S' or its number

Of elements are denoted by n(S).

Example! —
while rolling a die, the no that would appear would be any of the numbers among 1, 2, 3, 4, 5, 6 so here!-

$S = \{1, 2, 3, 4, 5, 6\}$ and $n(S) = 6$

Similarly in the case of tossing an unbaised coin

$S = \{$Head, Tail$\}$ or $\{H, T\}$
and $n(S) = 2$

The elements of the sample space are Called. sample points.

Event ! —
Every subset of a sample space is an event it is denoted by 'E'.

Example! — in throwing a dice
$S = \{1, 2, 3, 4, 5, 6\}$ the appearance of an even no will be the event
$E = \{2, 4, 6\}$. clearly E is subset of S.

• Types of events ! —

PAGE NO.
DATE:

- Types events :—

Simple event :- An event Consisting of a single sample Point is called a Simple event.

Compound event :— A subset of the sample space, which has more than one element is called a Compound event.

- equally likely events :—

Events are said to be equally likely if there is no reason to believe that One is more probable to Occur than the other.

-

Exhaustive Events :—

Two or more events are said to be exhaustive if they Collectively Constitute the sample space.

- Sure event :—

let 's' be a Sample space. if E equal to s the E is called a sure event.

- mutually exclusive or disjoint event :—

if two or more events Cannot Occurs simultaneously i.e. no two of them can Occur simultaneously., they are caused mutually exclusive or disjoint events.

PAGE NO.
DATE:

- Types events :—

Simple event :- An event Consisting of a single sample point is called a Simple event.

Compound event :— A subset of the sample space, which has more than one element is Called a Compound event.

- equally likely events :—

Events are said to be equally likely if there is no reason to believe that One is more probable to Occur than the Other.

- Exhaustive Events :—

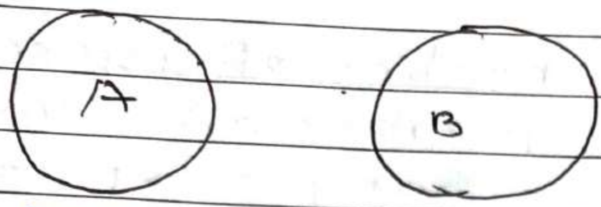
Two or more events are said to be exhaustive if they Collectively Constitute the sample space.

- Sure event :—

let 's' be a Sample space. if E equal to s the E is called a sure event.

- mutually exclusive or disjoint event :— if two or more events Cannot Occurs simultaneously i.e. no two of them can Occur simultaneously, they are caused mutually exclusive or dis-joint events.

PAGE NO.
DATE:



$A \cap B = \phi$

• Independant or motoally independent events :—

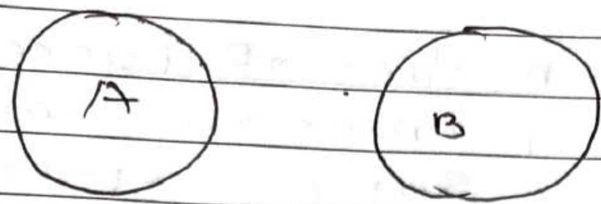
two or more are said to be independent if the accurance or non-Occurance of them does not affect the probability of Occurance or non-Occurence of the other events.

* Complement of an -event :—

let s be the Sample for random experiment and E be an event of Occurance of head in the second throw & the event of Occurance of head in the second throw are independent event.

• Classical defination of Probability :

if the 's' be the sample space then the probability of Occurence of an event 'E' defined as—



$A \cap B = \phi$

- Independant or mutoally independent events:—

two or more are said to be independent if the accurance or non-Occurance of them does not affect the probability of Occurance or non-Occurrence of the other events.

* Complement of an-event:—

let s be the Sample From random experiment and E be an event oF Occurance of head in the second throw & the event of Occurance of head in the second throw are independent event.

- Classical defination of Probability:

if the 's' be the sample space then the probability of Occurence of an event 'E' defined as—

PAGE NO.
DATE:

$P(E) = \frac{n(e)}{n(S)} = \frac{\text{number of elements in 'E'}}{\text{number of elements in sample space 'S'.}}$

• Properties of Probability :—

① The Probability of an event lies between '0' & '1'. i.e $0 \leq P(E) \leq 1$.

② The probability of an impossible event is '0' & e. $P(\phi) = 0$

③ The Probability of a sure event is 1 i.e. $P(S) = 1$, where 's' is the sure event.

④ if two events 'A' & B such that $A \subset B$ then $P(A) \leq P(B)$

⑤ IF 'E' is any event and 'E'' be the complement of event 'E', the $P(E) + P(E') = 1$

• Problems :—

Q1) → let 'E' =
Event of getting all heads
then $E1 = \{HHH\}$; $n(E1) = 1$
$P(E1) = \frac{n(E)}{n(S)} = \frac{1}{8} = 1/8$

PAGE NO.

DATE:

(ii) Let 'E_2' =
Event of getting at least ~~one~~ '2' head
then $E_2 = \{HHT, HTH, THH\}$;
$n(E_2) = 3$
$P(E_2) = \frac{3}{8}$

(iii) let 'E_3' =
Event of getting at least one head
then $E_3 = \{HHH, HHT, HTH, THH, HTT, THT, TTH\}$;
$n(E_3) = 7$
$P(E_3) = \frac{7}{8}$

(iv) let 'E_4' =
Event of getting at least two heads then $E_4 = \{HHH, HHT, HTH, THH\}$;
$n(E_4) = 4$

$P(E_4) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

Q2) → (i) P(1 knows, 1 queen, 1 king, 1 ace)

$= \frac{{}^4C_1 \times {}^4C_1 \times {}^4C_1 \times {}^4C_1}{{}^{52}C_4}$

$= \frac{256}{270725} = 0.0009$

PAGE NO.

DATE:

ii) p (Four honours of same suit)

$$= \frac{1+1+1+1}{{}^{52}C_4} = \frac{4}{270725}$$

iii) p (one from each suit)

$$= \frac{{}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_4} = \frac{71}{673}$$

$$= 0.1055.$$

iv) p (2 red, 2 black) $= \dfrac{{}^{26}C_2 \times {}^{26}C_2}{{}^{52}C_4}$

$$= \frac{325}{833} = 0.3902.$$

10/05/[illegible]

Notes

provided

by

Vaibhavi Jadhav